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ABSTRACT

This document reports on findings for Ontario from the Second International Mathematics Study (SIMS), planned as a documentation and analysis of mathematics education. Mathematics achievement was only one component: the overall objective was to learn about the teaching of mathematics, what was taught, how it was taught, and what methods were most successful. Chapter 1 reviews the design of SIMS and describes how different interpretations of the two populations to be studied led to quite different samples of students from country to country. The second chapter is concerned with the officially mandated curriculum. Characteristics of Ontario schools and teachers are described in this chapter and compared with schools and teachers in other countries. Ontario student achievement and attitudes are examined in chapter three. Achievement is discussed topic by topic in relation to other characteristics, with emphasis on students' opportunity to learn; gender differences are also explored. Attitudes are regarded as an outcome in their own right as well as in relation to achievement. The final chapter presents some of the lessons learned and plausible conclusions from the study. Results from Ontario are discussed mainly in the international context. A list of SIMS participants and a list of reports published or to be published on SIMS are appended. (MNS)

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RESEARCH BRIEF

LEARNING ABOUT TEACHING FROM COMPARATIVE STUDIES

Ontario Mathematics in International Perspective

LES MCLEAN
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MERLIN WAHLSTROM

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SE 047 721

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Preface

Large cross-national surveys of educational achievement take a decade to plan and carry out, involve thousands of people and cost millions of dollars. Why do people do it? What benefits could possibly justify such an effort? This report is one attempt to answer these questions, an answer given in the context of one of the two Canadian provinces that participated as "countries" in the Second International Mathematics Study (SIMS). The project was carried out in the early 1980s under the aegis of the International Association for the Evaluation of Educational Achievement, better known as IEA. It was one of an ongoing series of such surveys, including mother tongue and foreign languages, science, classroom environment, school-to-work and item banking.

The first international mathematics study was completed in the 60s and the results reported in two volumes (Husén, 1967). That study concentrated on school organization, including curriculum, using achievement in mathematics as a general indicator of school success. It was not a study of mathematics education, but rather a study of schools and schooling, using mathematics achievement as an indicator of output.

From the beginning, the second study (SIMS, as it came to be known) was planned as a documentation and analysis of mathematics education. Mathematics achievement was only one component, albeit an important one, of the overall picture. The objective was to learn about the teaching of mathematics--what was taught, how it was taught and what methods were most successful. By comparing and contrasting the findings from small and large countries around the world, both developed and developing, greater understanding can be reached than if studies are done separately in one country at a time. School organization was documented and social class measured as correlates of the curriculum and teaching methods as well as of achievement.

Two important transition points were chosen for the mathematics education study--the earlier one at the end of elementary school or the very beginning of secondary and the later one at the end of secondary school. The sample at the earlier point, *Population A* for short, was to be taken from all students in the grade in which the largest number had attained the age of 13.0 - 13.11 years of age by the middle of the school year. In most countries, all children are still in school at this age. In contrast, the sample at the later point included only students who were specializing in mathematics--those who were studying mathematics for five or more hours per week. By this time, the proportion of young people still in school is under 10 percent of the age group in some countries. A list of participating countries and the abbreviations to be used for country names is given in the display below. More information on participating countries is presented in Appendix A.

As this little book is being written, most of the participating countries have published their "national" reports, and the results of the international analyses are in press. Numerous topical reports and several overviews have already appeared in teachers' publications and in academic journals.[1]

Chapter 1 reviews the design of SIMS and describes how different interpretations of the two populations led to quite different samples of students from country to country. Ontario's Population A

Countries Participating in the Second International Mathematics Study
and Abbreviations Used in This Report

Belgium, Flemish schools (BFL)	Israel (ISR)
Belgium, French schools (BFR)	Japan (JPN)
Canada, B.C. (CBC)	Luxembourg (LUX)
Canada, Ontario (CON)	Netherlands (NTH)
England and Wales (ENW)	New Zealand (NZE)
Finland (FIN)	Scotland (SCO)
France (FRA)	Swaziland (SWA)
Hong Kong (HKO)	Sweden (SWE)
Hungary (HUN)	Thailand (THA)
Ireland (IRE)	United States (USA)

sample (taken from Grade 8 classes) was more varied than most and more representative than British Columbia's, for example. The older group (Population B) represented a wider segment of the Ontario population than in most countries, even though the definition limited the group to students taking at least two of the three advanced mathematics classes at the Grade 13 level. Ontario's diverse population also showed itself in an unusual range of both parents' occupation and education.

The second chapter is concerned with the officially mandated curriculum, that specified by the Ministry of Education in curriculum guidelines and with the assembly of items to test achievement of students who study within those curriculums. Ontario is not so different from the rest of the world. Teachers everywhere make many choices, however, within the guidelines, and their preferences result in important variations from school to school and from class to class in what is taught and what is emphasized. Characteristics of Ontario schools and teachers are described in this chapter and compared with schools and teachers in other countries.

Ontario student achievement and attitudes are examined in Chapter 3. By this time, Chapters 1 and 2 have made it clear that there are no simple, direct comparisons to be made among these different types of samples from different countries. This is true even between Canada (Ontario) and Canada (British Columbia)! Achievement is therefore discussed first in Ontario, topic by topic, often item by item, in relation to other characteristics. In view of the widespread interest in gender differences, achievement differences between boys and girls are explored. Everywhere, achievement is related to instructional emphasis, that is, to students' *opportunity to learn*. Attitudes are regarded as an outcome in their own right as well as a measure to be correlated with achievement. On average, the performance of Ontario students placed them at the middle of participating countries, Ontario often defining the exact median among countries.

The final chapter is a presentation of some of the lessons learned and plausible conclusions from the study. Results from Ontario are discussed mainly in the international context. Given that the international volumes are not even published yet, it is too soon for any summary of conclusions. That will take still more study and perhaps further analyses. It does seem that Ontario offers less mathematics in Grade 8 than some other countries, but more mathematics is offered to more students in Grades 12 and 13. Many countries offer more algebra in Grade 8, for example, and students show they can learn the content. Lots of time is spent everywhere reviewing old arithmetic content, but no systematic gain in achievement results from it. Most countries channel students into several tracks by the time they reach Grade 8, Ontario, France and Japan being exceptions.

The amount of homework done varies a great deal from country to country. Students who do more homework tend to achieve slightly higher scores, as most people expected. There may be some question whether the amount of calculus learned justifies the amount of time spent on it. Ontario has a high rate of student participation in Population B, compared to other countries, and the top students compare very well with top students elsewhere. In the end, however, there is no one conclusion. There are many conclusions and much food for thought.

Acknowledgements

Funds for the Second International Mathematics Study (SIMS) were provided to the Ontario Institute for Studies in Education by the Ontario Ministry of Education. Ministry staff served on the Advisory Committee and provided help and support throughout the project. Dr. L.D. McLean was National Research Coordinator, Dr. M.W. Wahlstrom was co-Coordinator, and Dr. Dennis Raphael was the full-time manager for this complex undertaking. Essential advice and support were provided throughout by Richard Wolfe, who also served as consultant to the International Mathematics Committee of SIMS. These people and others too numerous to mention made it possible for Ontario to participate in this important study.

The present report owes much to the drafts of the international volumes--so much that it is impossible to refer to the volumes at each point. Wherever Ontario results are compared with those of other countries, the comparisons were derived from the manuscripts of forthcoming international reports. Thanks must go to Ken Travers, Bob Garden, Daniel Robin, David Robitaille and the other authors. Appendix B lists all reports currently in preparation, with projected publishers and publication dates.

Dr. H. Howard Russell, who had been in contact with the organizers from the beginning of SIMS, chaired the Advisory Committee that guided the Ontario study:

Professor Ed Barbeau, U. of Toronto
Lorna Morrow, North York Board
Norman Sharp, Etobicoke Board
Robert Tuck, Nipissing Board
Jerry Morgan, Dufferin-Peel RCSSB
Maurice Poirier, Carleton Board
Michael Silbert, Hamilton Board
Les McLean, OISE

Chapter 1
The Many Faces of Mathematics Education

The first international mathematics study was a pioneering effort and the reports have been widely cited. The study was also widely criticized, however, especially in the mathematics and mathematics education communities.[2] A particularly lengthy and trenchant criticism came from a professor in the Netherlands who was at the time the editor of a respected mathematics education journal (Freudenthal, 1975). He attacked virtually every aspect of the study and noted that mathematicians and mathematics educators had not been centrally involved in its planning or implementation.

When a group met in Scotland in 1974 to consider undertaking the second study, they resolved that both mathematicians and mathematics educators would be given prominent place. The first international meeting to consider details was held at the University of Illinois in 1976, where they began to design a study of mathematics education that would document what was being taught and how it was being taught in addition to what was learned. In addition, there would be a thorough survey of school system, school, teacher and student characteristics that would enable interpretation of the teaching and learning findings in a wide context. Teacher and student attitudes would also be recorded in detail, all to be done at the end of elementary school (*Population A*) and at the end of secondary school (*Population B*) to fill out the most complete picture of teaching and learning yet attempted for any subject.

1.1. Describing the Content and the People

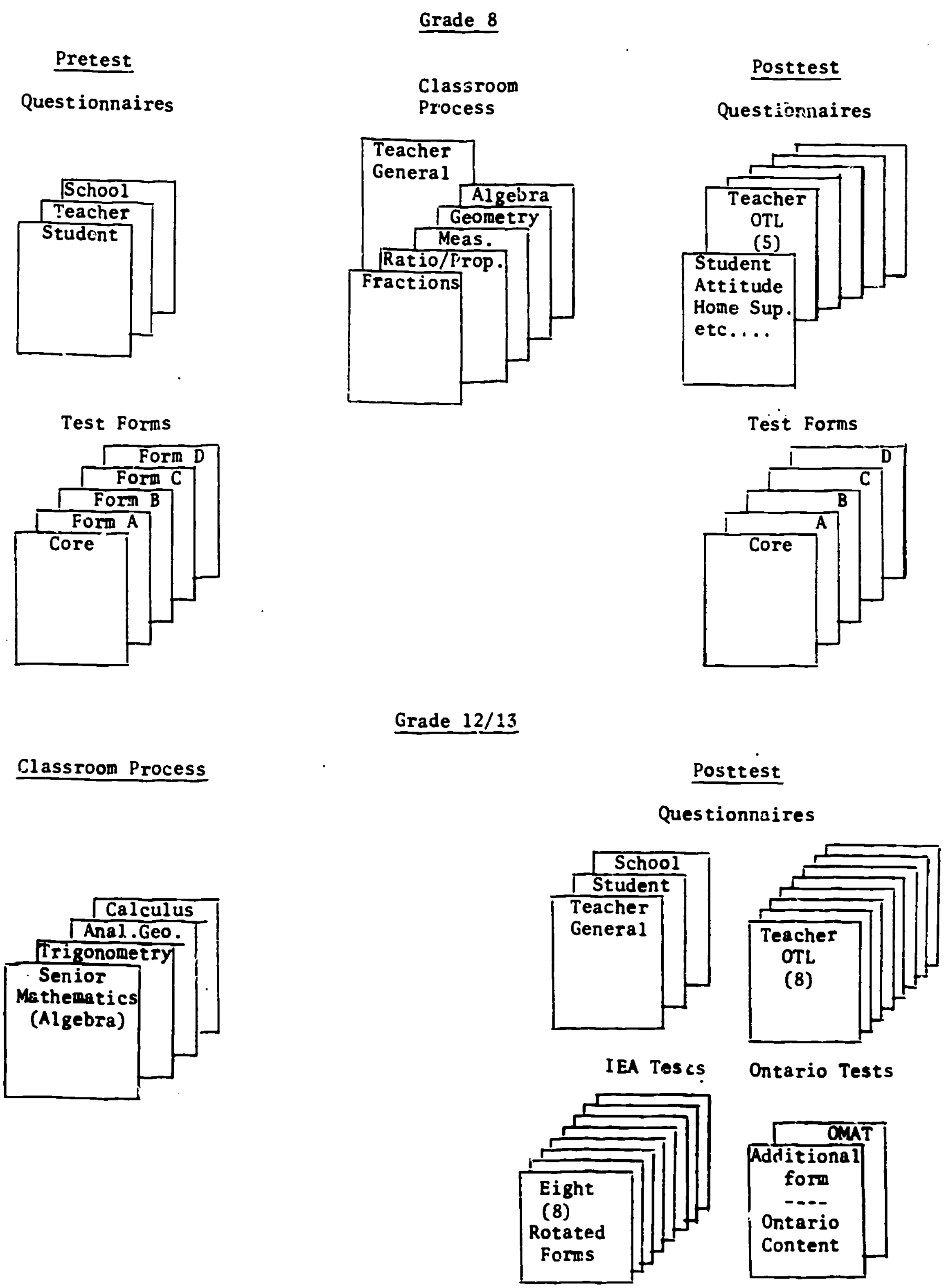
In order for SIMS to be a study of mathematics education and not simply a survey of student achievement, information was collected on just about every aspect of teaching and learning that was amenable to study by testing, questionnaires and analysis of documents. Provision was made for achievement testing at the beginning and at the end of the school year, referred to as the pretest and the posttest. Only eight countries administered both pretest and posttest in Population A, however, and only one (the USA) administered both in Population B. A whole series of questionnaires were developed for the other dimensions of the study, designed for administration at several times during the school year. The various forms and test booklets and their timing are illustrated in Figure 1-1.

In Figure 1-1, the questionnaires and test forms listed under *Pretest* were the ones administered at the beginning of the year in Grade 8. There was no pretest in Ontario in Grades 12 and 13. The forms listed under *Classroom Process* were completed by most teachers just after they finished teaching the content. The detail in the classroom process questionnaires is illustrated in Figure 1-2 by a selection of teaching methods explored in the *Fractions* form. It is likely that some teachers would not have heard of all the possible ways to teach fractions, and indeed a few tended to dominate. Each such form asked about use of teaching materials, time spent on topics, teaching methods, interpretations of important concepts and opinions about teaching the topic.

Figure 1-1: School, teacher and student questionnaires and test booklets used with Populations A and B in Ontario's implementation of the second international mathematics study.

INSTRUMENTS

SECOND INTERNATIONAL MATHEMATICS STUDY

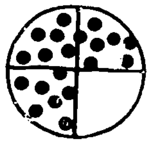


for the topic fractions

Teaching Methods - Common Fractions

The interpretations of fractions given below may be included in your instructional program.

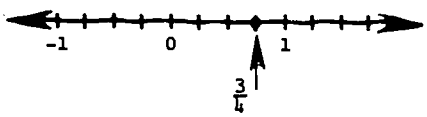
Fractions as parts of regions:

$\frac{3}{4}$ means 

Fractions as parts of a collection:

$\frac{3}{4}$ means 

Fractions as the coordinates of points on a number line:



Fractions as quotients:

$\frac{3}{4}$ means "3 divided by 4"

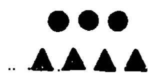
Fractions as decimals:

$\frac{3}{4} = 0.75$

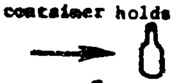
Fractions as repeated addition of a unit fraction:

$\frac{3}{4} = \frac{1}{4} + \frac{1}{4} + \frac{1}{4}$

Fractions as ratios:

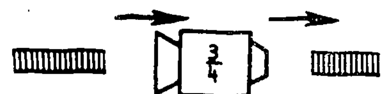
$\frac{3}{4}$ means 

Fractions as measurements: this container holds

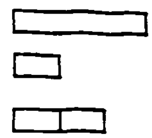
$\frac{3}{4}$ L 

this stick is $\frac{3}{4}$ cm 

Fractions as operators:



Fractions as comparisons:



unit rod

$\frac{1}{3}$ rod

$\frac{2}{3}$ rod

Two $\frac{1}{3}$ rods = a $\frac{2}{3}$ rod

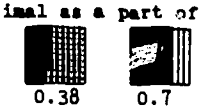
A decimal as the coordinate of a point on the number line.

0.28 < 0.8

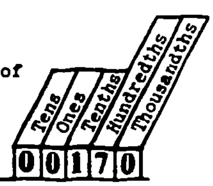
A decimal as another way of writing a fraction.

$0.17 = \frac{17}{100}$ $\quad 0.8 = \frac{8}{10}$

A decimal as a part of a region.



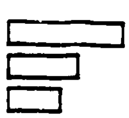
A decimal as an extension of place value.



A decimal as a series.

$0.245 = \frac{2}{10} + \frac{4}{100} + \frac{5}{1000}$

A decimal as a comparison.



Unit rod

0.6

0.45

Relate operations with decimals to operations with fractions.

Ex: $0.7 \times 0.6 = \square$

But $0.7 = \frac{7}{10}$ and $0.6 = \frac{6}{10}$

So $0.7 \times 0.6 = \frac{7}{10} \times \frac{6}{10}$

$= \frac{42}{100}$ Therefore $0.7 \times 0.6 = 0.42$

Relate operations with decimals to operations with whole numbers, teaching rules for placing the decimal point.

Ex: $1.38 \times 5.2 = \square$

Since
$$\begin{array}{r} 1.38 \\ \times\ 5.2 \\ \hline 276 \\ 690 \\ \hline 7176 \end{array}$$

$\underbrace{1.38}_{\textit{2 places}} \times \underbrace{5.2}_{\textit{1 place}} = \underbrace{7.176}_{\textit{3 places}}$

Use concrete materials to illustrate operations with decimals.

Ex: $3.47 + 2.13 = \square$

Using rods or match sticks, I demonstrated that

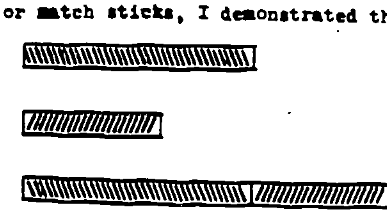
3.47 =

and

2.13 =

makes

5.60 =



The achievement items (more about these later) were divided over several test forms, because there were far too many to ask students to answer them all. Each student answered the items in two forms. All items, however, were reproduced in the *Teacher OTL* booklets, where teachers were asked whether the students were likely to answer the item correctly and whether they had taught the material (see Section 2.5). Other forms contained questions about the schools, about students' and teachers' background and attitudes, and all this information was supplemented by an extensive analysis of the curriculum carried out by staff at the national centres and experienced mathematics educators. The pool of achievement items and the innovative questionnaires were some of the many useful residual benefits countries retained after participating in SIMS.

1.1.1. Curriculum and Teachers

If the test items were to reflect the mathematics being taught, the first step had to be a curriculum survey. Since most countries have a national curriculum, documentation of this national curriculum was interesting by itself. Everyone recognized, however, that what was actually taught did not always correspond to what was intended. Moreover, students seemed to know some mathematics that they were apparently not ever taught and, more often, to be unaware of topics teachers were sure had been taught. Thus was born the overall design concept sketched in Figure 1-3, that there were actually three curriculums everywhere--the official, or *intended* curriculum, the actually taught, or *implemented* curriculum, and finally what students learned, the *achieved* curriculum. Chapter 2 is devoted to the curriculum study and teacher survey.

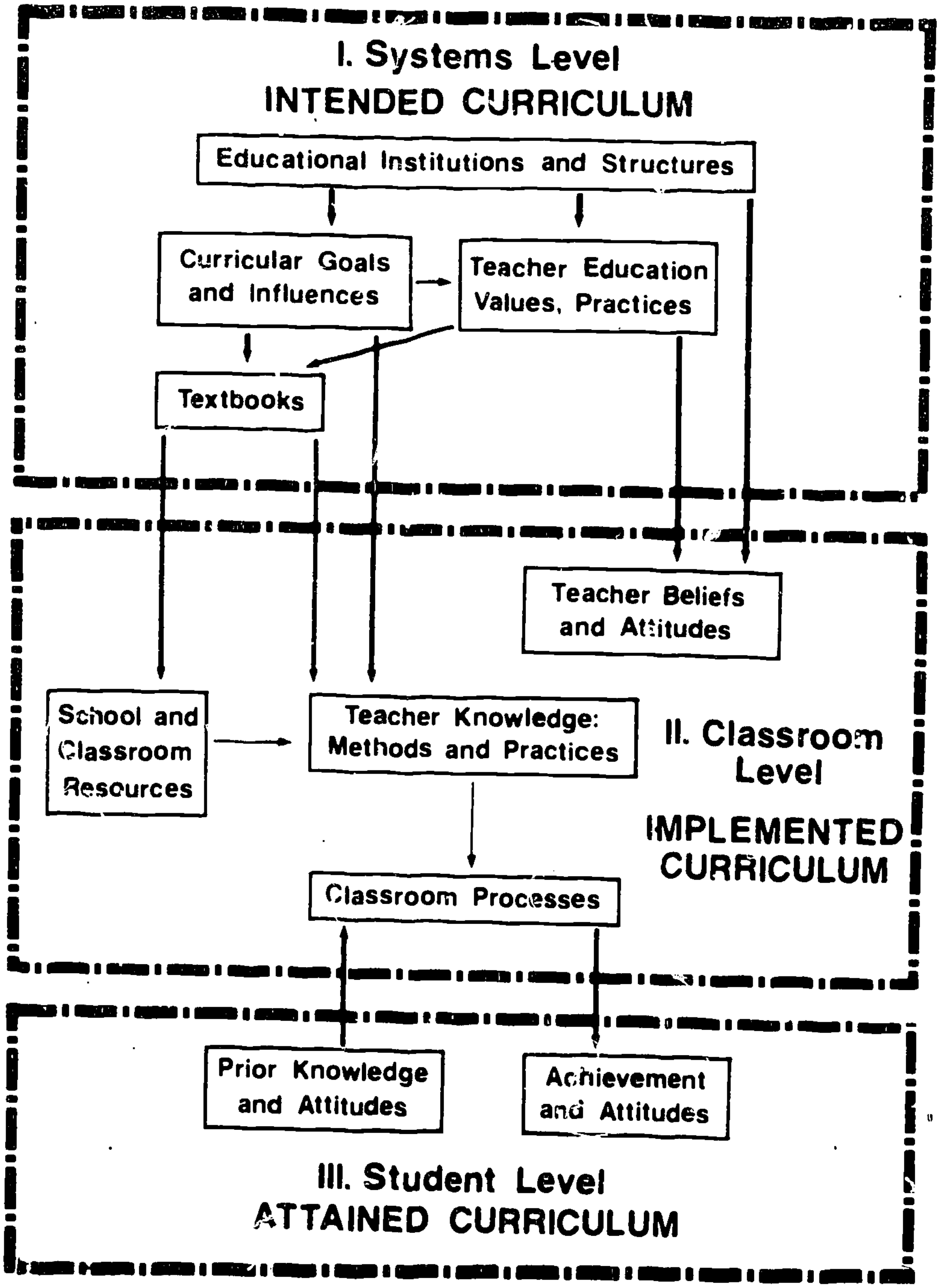
In almost every respect, Ontario Grade 8 teachers were similar to those in other participating countries. Teachers everywhere were experienced (12 to 16 years, except Thailand), their average age was 36 (40 in Japan) and there were more males than females (except Thailand). The striking exception was their degree of specialization in mathematics. In every other country, over 80 percent of mathematics teachers specialized in teaching the subject, while in Ontario the figure was 38 percent (either a Type A or Type B Mathematics Certificate or an Intermediate Certificate with a mathematics option). Teachers in other countries reported spending most of their time teaching mathematics, while Ontario Grade 8 teachers listed many subjects. This difference was also reflected in the amount of mathematics studied during their preservice training--less in Ontario than elsewhere.

Teachers in most countries said that their mathematics classes were easy to teach. The exceptions were Thailand and Japan, where 30 to 40 percent of the teachers reported that mathematics was a difficult subject for them. At the extremes, over 80 percent of the teachers in Canada, New Zealand and the USA felt that mathematics was easy to teach, but only 8 percent of Japanese teachers felt that way. (The Japanese students echoed the opinions of their teachers, reporting on their questionnaires that mathematics was more difficult and less enjoyable in comparison with students in other countries.)

When asked what factors were most responsible for lack of satisfactory progress in learning mathematics, teachers were most likely to find fault with the students. Student lack of ability or motivation were the causes most often given. Possible causes seldom chosen on the questionnaires included misbehaviour, fear of math (chosen by some teachers in France), absenteeism (mentioned in British Columbia), and lack of resources (chosen more often by Thai teachers). Poor teaching was chosen only by the Japanese teachers as a possible reason why students failed to make satisfactory progress.

Teachers everywhere have their own view of their subject, what is most important to emphasize--the overall objectives. Examples would be, *understanding the nature of proof* and *learning to perform computations with speed and accuracy*. Where the first is emphasized, it marks a more formal approach,

Figure 1-3: An expanded model for the three curriculums in the design of the second international mathematics study



one that is not recommended for Grade 8 in Ontario. The second represents an objective pursued to some extent everywhere, so it is instructive to know how much importance teachers give to it *relative to other objectives*.

Nine objectives were presented in one questionnaire, and teachers were asked to rate the relative emphasis they were giving to each one on a three-point scale. Which nine are selected for inclusion in the questionnaire is therefore quite important, since teachers might regard an objective not listed as more important than any of those on the list. The relative emphasis they give to each one will be influenced by the choice of objectives with which they are to compare. Teachers do have different views of mathematics, since no one of the objectives was consistently emphasized in all countries. Responses were summarized by calculating the percentage of teachers in each country who felt that a given objective should receive more emphasis than was accorded to others in the list.

The nine objectives selected (with percentage of Ontario teachers in parentheses) were:

1. Understand the logical structure of mathematics. (39%)
2. Understand the nature of proof. (11%)
3. Become interested in mathematics. (39%)
4. Know mathematical facts, principles, and algorithms. (30%)
5. Develop an attitude of inquiry. (43%)
6. Develop an awareness of the importance of mathematics in everyday life. (58%)
7. Perform computations with speed and accuracy. (35%)
8. Develop an awareness of the importance of mathematics in the basic and applied sciences. (19%)
9. Develop a systematic approach to solving problems. (63%)

Objective 2 was given low ratings by all countries except France and New Zealand, suggesting that teachers in these countries take a more formal view of mathematics at this level. Objective 8 was rated low by everyone, indicating, one assumes, that this understanding, however important, will have to come later. Objective 6 was favoured by a majority of teachers only in Ontario and the USA. More teachers in British Columbia gave high ratings to objective 9 than any other country, and teachers in France were very much more in favour of developing an attitude of inquiry (objective 5).

One would not attempt to draw many conclusions from these responses to abstract statements of objectives if they were the only evidence available. In the context of SIMS, however, they are quite consistent with other evidence for great diversity among countries in the way the teachers construe the content of the mathematics curriculum. Teachers agree on many topics, but how they approach those topics, what emphasis they give to them, appears to vary considerably both between and among countries.

To probe these perceptions in another way, teachers were presented 15 statements about the nature of mathematics and asked to respond on a five-point scale, from Strongly Disagree to Strongly Agree. The 15 items had been shown in pretesting to form a scale that was named *Mathematics as a Process*. A high score signals a teacher who sees mathematics as a field that is changing, growing and developing in new directions, while a low score suggests a view of mathematics as static, completely determined and unchanging. The items are listed in Table 1-1.

There were no differences among countries in scores calculated from the 15 items as a whole. There were consistent differences in responses to some individual items, however. Very few teachers disagreed with statement 5, *Mathematics helps one think according to strict rules*. On the other hand, there was overwhelming agreement with statements 6, 7, and 15 (and disagreement with 8). Overall, teachers everywhere gave modest support for a process view rather than a static view of mathematics. Ontario and

Table 1-1: Items Making Up the Scale, *Mathematics as a Process*

1. Mathematics will change rapidly in the near future. (63%)
2. Mathematics is a good field for creative people. (50%)
3. There is little place for originality in solving math problems.* (28%)
4. New discoveries in mathematics are constantly being made. (52%)
5. Mathematics helps one think according to strict rules.* (76%)
6. Estimating is an important mathematics skill. (86%)
7. There are many different ways to solve most math problems. (78%)
8. Learning mathematics involves mostly memorizing.* (15%)
9. In mathematics, problems can be solved without using rules. (44%)
10. Trial and error can often be used to solve a math problem. (75%)
11. There is always a rule to follow in solving a math problem.* (55%)
12. There have not been any new discoveries in math for a long time.* (32%)
13. Mathematics is a set of rules.* (45%)
14. A mathematics problem can be solved in different ways. (65%)
15. Mathematics helps one to think logically. (94%)

* = Item marked in reverse order to others in calculating scale score.

The numbers in parentheses are the percentage of Ontario teachers who agreed with each item.

Thai teachers were more in agreement than the others that mathematics will change rapidly in the near future. The Flemish in Belgium held views opposite to just about everyone else, especially the Japanese, about the usefulness of trial and error in solving mathematics problems. The Belgian teachers, on the Flemish side at least, said it was not useful, while the Japanese teachers said it could be very useful.

When we come to the ways teachers allocate their time among their various duties, the diversity disappears. As can be seen in Table 1-2, there is little variation among countries in the reports teachers give of the time they spend preparing lessons, grading papers and the like. They also report about the same proportions spent explaining new content, reviewing and testing. They said they spent little time disciplining and administering and lots of time explaining, reviewing and testing. In another section of the questionnaire they also reported that most of their time was used in whole-class instruction rather than small-group or individual work.

1.1.2. Schools and Students

The first mathematics study had an elaborate plan for choosing participating schools, teachers and students with several populations and subpopulations. The plan was rarely implemented and drew numerous criticisms that the SIMS planners were determined to avoid. Only two populations were selected for the second study, (a) the grade in which the modal age is 13, and (b) mathematics specialists, students in the last year of secondary education who are taking advanced mathematics as a substantial (five hours per week) part of a program leading to post-secondary education. Countries were urged to select two classes per school, wherever possible. Strict sampling rules were laid down and supervised by an international sampling referee. In spite of these efforts, the resulting samples differed in ways that are important to note when attempting any comparisons of results from country to country.

Table 1-2: Teacher Reports of Allocation of Teaching Time among Their Various Duties

	BFL	CBC	CON	FRA	JPN	NZE	THA	USA
Preparation (min/week)	80	80	60	90	90	40	60	60
Grading papers (min/week)	90	60	60	100	60	30	180	80
Explaining (%)	50	26	30	27	33	19	45	36
Reviewing (%)	25	17	25	23	22	12	20	20
Administering (%)	15	6	10	4	6	5	10	10
Disciplining (%)	2	4	5	2	6	5	8	4
Testing (%)	11	17	15	22	11	6	15	18

BFL: Belgium/Flemish; CBC: Canada/British Columbia; CON: Canada/Ontario;
FRA: France; JPN: Japan; NZE: New Zealand; THA: Thailand; USA: United States.

Virtually every characteristic of school showed considerable variation among countries. Table 1-3 lists the per school averages of four characteristics for the eight countries in the longitudinal study--enrolment, pupil-teacher ratio, length of math period and hours of math per year. Ontario is unusual for the relatively short math periods--40 minutes as compared with 50 minutes to one hour in other countries--and small schools. Ontario has one of the smallest average enrolments of any country, about the same as the Flemish schools in Belgium (but Ontario has a much higher pupil-teacher ratio). The total number of hours per year in Ontario equals or exceeds that in all but two other countries. The total number of hours per year is smallest in Japan, a surprising result in view of the good achievement there (achievement will be discussed in section 4.1).

Table 1-3: Characteristics of Schools Participating in the Population A Longitudinal Study

	BFL	CBC	CON	FRA	JPN	NZE	THA	USA
Average enrolment per school	366	575	375	610	714	848	1293	548
Pupil-teacher ratio	9.4	17	19.8	16.9	22	18.8	28.7	15.2
Length of math period	50	60	40	55	48	60	50	48
Hours of math per year	140	120	132	130	101	130	120	144

Recall that Ontario chose Grade 8 for its *Population A*. This is the grade where most 13-year-olds are found, but Grade 8 is also an interesting point for a summative assessment because it is also the normal last year of *elementary* school. British Columbia and the USA also chose Grade 8, but Japan sampled classes from Grade 1 of lower secondary school, equivalent to Ontario Grade 7. Other countries made modifications to the definition, with the result that the average age of participating students varied from 13.2 years to 14.8.[3] In May 1982, Ontario's students averaged 13.9 years of age and British Columbia's 14.0.

Students in SIMS were supposed to be a representative, random sample from their population, but this was not always achieved. In British Columbia, a careful random sample of schools was obtained, but only one class was chosen per school. Wherever there was more than one Grade 8 class in the school, the principal was allowed to choose the class (Robitaille, O'Shea, & Dirks, 1982). In Ontario, the SIMS team at OISE chose two classes at random from a list supplied by the principal, wherever there was more than one. As will be seen, the resulting sample in B.C. was unusual in several respects. In England and Wales a number of schools declined to participate, putting the representativeness of their sample in doubt, and the same was true in the USA. Similar problems in other countries may have gone undetected. The excellent cooperation in Ontario was encouraged by a memorandum from the Ministry of Education to directors of boards designating SIMS as an official study. Some details of the Population A samples are given in Table 1-4 for the eight countries that tested achievement both at the beginning and at the end of the school year.

Table 1-4: Characteristics of the Population A Samples for the Eight Longitudinal-Classroom Process Countries

Country	Number of Strata	Number of Schools	Number of Classrooms	Number of Students
Belgium (Flemish)	16	168	175	4519
British Columbia	6	90	93	2567
Ontario	24	130	197	6284
France	8	184	365	8778
Japan	19	210	211	7785
New Zealand	6	100	196	5978
Thailand	13	99	99	4030
U.S.A.	7	161	302	8372

Students were asked to name and also to describe their mother's and their father's occupation and to tell how much education their parents had. This information was used to derive an indicator of social and economic class, since such indicators have always been found to be highly correlated with student characteristics, including achievement. In comparison with other countries, Ontario's Population A had an unusually large proportion of fathers in the category, *semi-skilled and unskilled workers*.[4] British Columbia had the highest percentage of fathers in the *Professional* or *Managerial* category of any of the countries in the study, while Ontario was right at the median of that category. Mother's and father's education were consistent with the pattern of occupations.

The definition of Population B as mathematics specialists in the last year of secondary school resulted in at least as much diversity among country samples as in Population A. Only a few examples can be given here. In most of the world, there is an elite, university-bound group that fits this definition well, in Europe, Africa and the Far East, for example. In the USA and British Columbia, however, there are only 12 years of secondary school and the majority of students do not take calculus. Ontario and Scotland had both a 12-year and a 13-year system at the time the study was done, and samples of students were drawn from both for the study. For the international analyses, Ontario submitted only the sub-sample of *specialists*, whereas Scotland submitted results from all students. The percentage of fathers in B.C.'s Population B whose occupation was classified as *Professional or Managerial* was above the median, while the percentages in Ontario and Scotland were below the median.

A statistic often used for judging the comparability of Population B samples is *retention rate*, the percentage of eligible students still in school when the sample is taken.[5] Because these rates were used to help explain the Population B results in the first study, and because they are interesting in their own right, retention rates estimated from the second study are reproduced in Table 1-5.[6]

Table 1-5: Percentage of Population B Students in Relevant Age or Grade Groups for Each Country: 1981

Country	Age Group (Years)	Mathematics Retention Rate	Mathematics Participation Rate	General Retention Rate
Belgium (Flemish)	17	9-10	25-30	65
British Columbia	17	30	38	82
England & Wales	17	6	35	17
Finland	18	15	38	59
Hungary	17	50	100	50
Israel	17	6	10	60
Japan	17	12	13	92
New Zealand	17	11	67	17
Ontario	18	19	55	33
Scotland	16	18	42	43
Sweden	18	12	50	24
U.S.A.	17	13	12-15	82

Taken from Miller and Linn (1985). The second column is not always a simple product of the third and fourth columns because not all students in the fourth column were in the grades from which the Population B sample was drawn.

Relative to other countries, a high percentage of Ontario students specialize in mathematics. This is seen in Table 1-5 (column 2), where Ontario has the fourth highest percentage of age group in the Population B sample, even though the percentage of the age group still in school is lower than most. The high percentage of age group still in school in the USA and B.C. (column 4 of Table 1-5) reflects the high proportion of students continuing through Grade 12, but note that the percentage in Japan is even higher. These data are discussed again in Section 4.2.1.

Unlike the first study, the finding from the second study was that retention rates had little impact on achievement in comparison with other variables (Miller & Linn, 1985). This was so whether one looked at the top students (top 1 percent and top 5 percent on SIMS achievement subtests) or at the whole Population B (see Section 4.2.3). The conclusion was that the curriculum (particularly opportunity to learn) was the strongest factor, but no one factor was adequate to explain the variations in achievement.

1.2. Supporting Classroom Instruction

Students who spend more time on homework get higher marks on tests and do better in school generally. This was found in the first mathematics study and has been confirmed in many other studies since then (Raphael, Wahlstrom, & Wolfe, 1985). This might be because better students just do more homework or because teachers assign more homework to high achievers, but the association is certainly strong and widespread. A researcher in the USA pulled together results from many studies and concluded that homework can enable less able students to perform up to the level of their more able classmates. Teachers, however, tended to put lower demands on the less able students, widening the gap rather than narrowing it.[7]

In the second mathematics study, students were asked how much mathematics homework they did and how much homework they did in all subjects. The highest reports came from Thailand, Belgium, France and Hungary and the lowest from Sweden, England and Scotland. As was the case so often, Ontario was at the median (with the USA) with 2.6 hours of mathematics homework per week, half of the total homework done. This total agrees with reports from a large science survey done the next year (McLean, 1986). B.C. was just below the median. The total amount of homework reported by SIMS participants in the USA (5.2 hours per week) was higher than that found in the large study mentioned above (see Note 7). That study reported a total figure of four hours per week in the USA, just a little more than the average amount of television watched *every night*. Mathematics homework accounts for a large proportion of total homework everywhere. In three quarters of the countries, mathematics homework accounted for 35 to 50 percent of the total, with Nigeria (NGE) and Swaziland standing well out from the others at 79 and 80 percent.

Calculators were widely used, but by no means universally so. At the time SIMS was conducted, four-function calculators were used by 25 per cent or more of Population A students in 80 percent of the countries, either in the schools or at home. Almost 30 percent said that they were used for *recreational* purposes. The highest percentage use was in France and Ontario (67 and 68 percent), and the lowest in Nigeria and British Columbia (3 percent). Scientific calculators were used by most Population B students (over 90 percent in England and Sweden), with the exception of Hungary (18 percent) and Japan (27 percent). The low percentage reported by the Japanese appears odd, since they make or sell most of the scientific calculators in the world, and their high-achieving students said on their questionnaires that use of calculators and computers are necessary to mathematics learning. Computers were not widely used at either level, with only New Zealand, Sweden and the USA reporting use by as many as 20 percent of Population B students.

Unlike language, mathematics is not learned at home, from television, from cinema or in the street.[8] In a 1981 survey (McLean, 1982b, p. 9), only 4 percent of Grade 8 students reported that they received regular help with their mathematics lessons from any of their family members, and only 45 percent said they received help "once in a while". Girls received occasional help slightly more often than boys. The figure drops to 2 percent for advanced level courses in Grade 10. Roughly the same percentages were found in SIMS, with the same gender differential. Parents tell children that mathematics is important and that it is important to do well, but they do not (perhaps cannot) provide regular help with the mathematics lessons.

Out-of-school tutoring by persons other than family members is another way of supporting classroom instruction, and 10 to 20 percent of students at the Population A level receive one to two hours of tutoring each week in just about every country. Such tutoring is rare, however, in Sweden and in England and Wales. There are four countries, where it is quite common. In Japan, Nigeria, Swaziland and Thailand, more than half the students receive an average of three hours per week of extra assistance. The *juku*, a private tutoring class, has long been recognized as a feature of Japanese education, but the prevalence of such schools in the other three less developed countries was not so widely known. Japan, Finland and Hungary also have many such classes at the Population B level.

The Japanese juku are quite controversial, with almost all educators of the opinion that they contribute little or nothing to children's education (T. Sawada, personal communication). Teaching is done by parents and retired teachers, few of whom have any knowledge of the curriculum. A few juku do offer systematic preparation for the state examinations, apparently with some success. Most drill the students in traditional calculations and application of standard rules, work the students already get a great deal of in their regular classes.

Chapter 2

The Official Curriculums--More Commonality than Diversity

2.1. Documenting the Intended and Implemented Curriculums

In most countries, the *intended* curriculum is easy to document, because it is prescribed by the Ministry or the Department of Education. This is the case in Ontario, where the Ministry of Education issues curriculum guidelines that schools and teachers are obliged to follow. These documents are just that, however, ***guidelines***, and they specify many options at each grade level. Schools are expected to prepare a *course of study*, giving details of their decisions on options, and teachers make decisions from year to year and class to class. Diversity is therefore expected, but the extent of variation from school to school and class to class surprised most observers.

This variation was documented in several ways. First, there was a background questionnaire for all teachers, asking about their experience and their preparation for teaching in some detail and then asking them to report how many periods per week they taught common fractions, decimal fractions, ratio and proportion, percent, measurement, geometry and algebra (formulae and equations). Much more detailed reports were provided by teachers on *Classroom Process Questionnaires*, one questionnaire for each of the above topics and one about teaching in general. One page asking about the teaching of the multiplication of integers is displayed in Figure 2-1.[9]

2.2. Assembling Items and Matching to the Curriculums

As soon as a dozen countries agreed to participate, copies of the intended curriculums were obtained and ***international grids*** were prepared for Population A and Population B. These were the large survey equivalents of test blueprints--detailed outlines of the content of the curriculums. Mathematics educators in each country went over each grid and rated the topics on a scale from very important to not important. Combining these ratings, each topic was classified as Very Important, Important, Important in Some Systems or Not Important.

While these ratings were being collected and studied, the task of item writing and assembly was begun. Since textbooks are influential in determining what is taught, an analysis of the textbooks in common use in each country was made a part of the design and of the process of choosing achievement items. In order to make comparisons with the first international study, 35 items were retained from that study for SIMS Population A (Grade 8 in Ontario) and 18 for SIMS Population B (Grades 12 and 13). All items were of the multiple-choice variety, with five alternatives. Items were written as needed to cover all topics in the grids, circulated to countries and eventually approved by the International Mathematics Committee for inclusion in the study. Almost all were given three trials in eight of the countries as part of the selection process.

Figure 2-1: Questions for teachers about the way they taught multiplication of integers to the target class.

RESPONSE CODE

a. Emphasized (used as a primary method of development, referred to extensively or frequently).

b. Used, but not emphasized.

c. Not used.

70. Development by use of repeated addition.

I developed the concept of multiplication by appealing to repeated addition, e.g.,

$4 \times {}^{-}3 = {}^{-}3 + {}^{-}3 + {}^{-}3 + {}^{-}3 = {}^{-}12$

71. Development by the extension of properties of the whole number system.

I developed the concept of multiplication by using the commutative, associative, and distributive properties to justify the products, e.g.,

${}^{-}4 \times {}^{-}3 = \square$

$0 = 0 \times {}^{-}3$

$0 = ({}^{-}4 + {}^{+}4) \times {}^{-}3$

$0 = ({}^{-}4 \times {}^{-}3) + ({}^{+}4 \times {}^{-}3)$

$0 = ({}^{-}4 \times {}^{-}3) + {}^{-}12$

Hence $({}^{-}4 \times {}^{-}3)$ is the additive inverse of ${}^{-}12$.

$\therefore {}^{-}4 \times {}^{-}3 = {}^{+}12$

72. Development by use of physical situations.

I developed the concept of multiplication of integers by appealing to physical situations that might illustrate the product of positive and negative numbers.

Ex: A refrigerator is cooling at a rate of 4^0/min. Its thermometer is currently at 0^0. What will be its temperature 4 min from now?

73. Development by use of patterns.

I developed the concept of multiplication of integers by appealing to patterns of products.

Ex: ${}^{+}4 \times {}^{-}3 = {}^{-}12$

${}^{+}3 \times {}^{-}3 = {}^{-}9$

${}^{+}2 \times {}^{-}3 = {}^{-}6$

${}^{+}1 \times {}^{-}3 = {}^{-}3$

$0 \times {}^{-}3 = 0$

${}^{-}1 \times {}^{-}3 = {}^{+}3$

${}^{-}2 \times {}^{-}3 = {}^{+}6$

74. No development--students were given rules.

I did not develop the concept of multiplication of integers by using any of the above methods. Instead, I gave the students rules similar to the following:

If the signs are alike, the answer is positive.
If the signs are different, the answer is negative.
If either factor is zero, the answer is zero.

Items were prepared first in English and then translated into the language of the participating countries. Two independent translations were recommended, and problems with translation were reported to the central office of the study. Some items were dropped because no satisfactory version could be found for all countries. In the end, an initial pool of 480 items for Population A and 400 for Population B were winnowed to 199 and 136, respectively.[10] The distribution over achievement subsets is shown in Tables 2-1 and 2-2.

Table 2-1: Structure of the Final Population A Item Pool in the Second International Mathematics Study

POPULATION A

Achievement Subset	Longitudinal Version	Cross-Sectional Version	Common to Both Versions
Arithmetic	62 (34%)	46 (25%)	46 (29%)
Algebra	32 (18%)	40 (23%)	29 (18%)
Geometry	42 (23%)	48 (27%)	40 (25%)
Measurement	26 (14%)	24 (14%)	24 (15%)
Descriptive Statistics	18 (10%)	18 (10%)	18 (11%)
TOTAL	180*	176	157

*There were 199 items in the final pool. Countries participating in just the end-of-year testing used a *cross-sectional* subset of 176. So did Japan, but the other seven countries that tested at the beginning and end of the year used a *longitudinal* subset of 180 items. A total of 157 items were in both the cross-sectional and longitudinal subsets, and these 157 were used in some analyses--the studies of OTL, for example.

Table 2-2: Structure of the Final Population B Item Pool in the Second International Mathematics Study

POPULATION B

Sets, relations, functions	7 (5%)
Number systems	17 (13%)
Algebra	26 (19%)
Geometry	26 (19%)
Analysis	46 (34%)
Probability and Statistics	7 (5%)
Not in subtests*	7 (5%)
TOTAL	136

*3 combinatorics items, 3 low coverage, 1 found flawed

How well the SIMS item pool matched a system's intended curriculum was measured by calculating the percentage of items in each topic subset that educators said were either *highly appropriate* or *acceptable* to that system. In other words, the index of *coverage* represents the proportion of items in the SIMS pool that are based on mathematics intended to be covered, as reported by experienced mathematics educators.

2.3. Intended Curriculum Content: Grade 8

Figure 2-2 summarizes the indices of intended Population A content coverage for those educational systems for which appropriateness ratings were available. Ontario intends to teach a higher percentage of the SIMS items in arithmetic, geometry, statistics and measurement but not so many in algebra. By inspection of the table it can be seen that education systems are most alike for arithmetic and measurement and most different for geometry and statistics. Given that the international intended curriculum was perceived as adequate to cover most mathematics programs, Ontario's overall high rank on intended coverage (sixth among countries) reinforces the relevance of the SIMS item pool to Ontario schools and encourages continued analysis.

Figure 2-2: Percentage of SIMS Population A items countries *intend* to cover, by topic, for each country.

Percent of Items Covered	Arithmetic	Algebra	Geometry	Statistics	Measurement
100	CON SCO USA	IRE NZE		NZE SCO SWE USA	CON ENW HUN ISR JPN NZE SCO THA USA
90	BFL BFR HKO ISR HUN IRE LUX NZE JPN THA CBC ENW	FRA HUN SCO BFL BFR JPN	JPN NZE SCO	CBC	FIN HKO IRE CBC SWE
80	FIN FRA SWA SWE NTH	CBC ENW NTH	ENW CON HUN	ENW FIN HKO HUN CON SWA	NTH SWA
70		ISR THA SWE LUX FIN HKO SWA	FIN ISR THA NTH	JPN THA	LUX
60		USA CON	SWA HKO	NTH IRE	FRA BFL BFR
50			SWE USA		
40			CBC FRA		
30			IRE		
20			BFL BFR LUX	LUX FRA	
10				BFL BFR	
0				ISR	
N.items	62	42	51	18	26
Avg.Pct.	92	83	64	69	91

Having the highest intended coverage for arithmetic may not be the most desirable outcome, however. As will be discussed in Chapter 4, it might be more appropriate to transfer part of this effort to algebra. The issue with geometry is different. Although the SIMS items were rated highly appropriate, many teachers simply did not get around to teaching geometry or gave it low priority for instructional time. Teachers rated fewer algebra items as appropriate or acceptable, because the topic is seen as one more suited to the secondary school. The clear exception was *integers*, included under algebra in SIMS and very widely taught in Grade 8.

2.4. Intended Curriculum Content: Grade 13 Specialists

Provincial data were gathered and reported in the Ontario national report (McLean, Raphael, & Wahlstrom, 1986) for Grade 12 and for all three Grade 13 courses, but because of the narrow definition of mathematics specialists (students taking two or three of the Grade 13 courses) the Population B intended coverage was judged only against the Grade 13 courses. The summary of intended curriculum coverage is presented in Figure 2-3.

Figure 2-3: Percentage of SIMS Population B items countries *intend* to cover, by topic, for each country.

Percent of Items Covered	Sets, Rel and Func	Algebra	Geometry	Elementary Function and Calc.	Prob. and Stat.	Finite Math
100	BFL BFR CBC CON FIN FRA HKO ISR LUX NZE SCO	BFL BFR CON ENW HKO IRE NZE SCO THA		NZE	CON ENW FIN FRA IRE JPN NZE	CON FIN FRA HKO IRE JPN LUX NZE SCO THA
90		FRA HUN JPN LUX SWE FIN ISR	HKO	FRA IRE LUX SCO BFL BFR FIN HUN ENW HKO ISR JPN USA		
80	IRE JPN THA USA	CBC USA	CON JPN IRE FRA LUX	CON SWE	HKO ISR LUX SWE THA USA	
70	ENW HUN SWE		FIN HUN ENW NZE			BFL BFR ENW SWE
60			ISR SCO THA SWE BFL BFR USA			
50			CBC THA	THA		USA
40					SCO	
30				CBC		
20					BFL BFR HUN	CBC
10					CBC	
0						HUN I?R
N. Items	7	25	28	46	7	4
Avg. Pct.	92	96	73	89	76	76

Eighty percent or more of the mathematics necessary to answer the algebra items was intend taught in all educational systems, whereas many more of the geometry items were not thougł appropriate or acceptable. Ontario and Japan rated 80 percent of the geometry items as appr whereas British Columbia and Thailand reported only 50 percent of them as appropriate. The two elementary functions and calculus and geometry were least well covered in Ontario in comparison educational systems, but the coverage was still 80 percent. British Columbia and the USA often low coverage because their systems only went to Grade 12. Not surprisingly, there was the variation in coverage of finite mathematics, a topic that has still not achieved high status i countries in spite of its many applications in fields such as computing.

2.5. Implemented Curriculum Content

One of the important objects of study in all recent IEA projects has been the variable descr "opportunity-to-learn" (OTL). The SIMS design asked classroom teachers to complete the " Opportunity-to-Learn Questionnaire" in which they were asked the following questions about each the student test forms used in the classroom:

1. What percentage of the students from the target class do you estimate will get the item corre without guessing?
   - Virtually none
   - 6-40 percent
   - 41-60 percent
   - 61-90 percent
   - Virtually all
2. During this school year, did you teach or review the mathematics needed to answer the ite correctly?
   - yes
   - no
3. If, in this year, you did NOT teach or review the mathematics needed to answer this ite correctly, was it because,
   - It has been taught prior to this school year?
   - It will be taught later (this year or later)?
   - It is not on the school curriculum at all?
   - For other reasons.

Students were also asked to answer for each item whether they had been taught the material year, (b) before this year or (c) not at all. The index derived from these responses is called *student* distinguish it from the one derived from teacher questionnaires, *teacher OTL*. The OTL variabl included to measure the extent of the *implemented curriculum*, but as of this writing only the teach data have been analyzed. Ontario administered the Teacher Opportunity-to-Learn Questionnaire grade levels, one of 16 systems to do so for Population A and one of 12 systems for Population B.

Teacher OTL taps not only the curriculum of the target year but also the background knowle teachers believe students bring to the testing situation. A difficulty with this measure is that t sometimes do not know what has been taught in earlier grades, and considerable difference of emerged when referring to the three parallel Grade 13 courses. Teachers were asked to choose one the *target class* and to answer all questions with reference to that class, but this may not always ha done. (The problem of rating OTL when there were simultaneous courses was also severe in New and in England and Wales.) In subsequent studies at the school board level in Ontario, the OTL questionnaires were completed by the mathematics department as a unit rather than by individual teachers. This procedure assured information that reflected OTL for the school, and the school was used as the unit of analysis, but there is still the problem of knowing which students had what *opportunity to learn.*

Figure 2-4 displays the percentages of SIMS items reported by teachers as covered in each country, for five topic areas. Canada/Ontario (CON), is moderately above the mean in all areas, the lowest percentage (50 percent) being reported in geometry. Thus, the pattern observed in the measures of intended curriculum are repeated in the implemented curriculum. This is encouraging, because the indices of intended curriculum were obtained from a few people rating the whole system, whereas the indices of implemented curriculum are averages over many teachers. For geometry, the primary variable influencing the amount of instruction appears to be cultural. Countries whose curriculum is modeled on the traditional English curriculum together with Hungary, Japan, the Netherlands and Thailand give geometry moderate or substantial emphasis and treat a more or less common core of subjects in plane figures and coordinates. The remaining countries--Ontario, British Columbia and the United States--devote little attention to the overall area and the topics covered are scattered.

Figure 2-4: Percentage of SIMS Population A items covered, by topic, for each country.

Percent of Items Covered	Arithmetic	Algebra	Geometry	Statistics	Measurement
90	FRA SWE JPN HUN	HUN	HUN		
80	LUX BFL CON NTH THA	NTH USA JPN SWE THA CBC FRA CON	THA CBC JPN FRA SWE	SWE HUN	SWE HUN USA JPN
70	FIN NZE NGE USA CBC ENW	ISR FIN BFL ENW LUX NGE	CON FIN NTH NGE BFL ISR		USA JPN
60	ISR SWE	SWE NZE	ENW NZE USA	NZE NGE NTH	CON NZE NGE ENW
50			LUX	CBC CON JPN ENW THA	FIN FRA ISR THA
40			SWE	ISR FRA USA	SWE CBC
30			FIN	BFL LUX SWE	NTH LUX BFL
20					
10					
N. Items	24	46	30	39	18
Avg. Pct.	73	73	67	45	51

Percentages of Population B items reported by teachers as covered are displayed in Figure 2-5. Ontario teachers cover about 80 percent of the algebra items in the SIMS pool. Ontario Grade 13 algebra appears to be unique in the topics covered when contrasted with other pre-university algebra courses. In the case of Ontario's calculus, figures not shown here reveal that there is variability of coverage but that about one half of the classes cover most of the SIMS domain. In Israel and in England and Wales, comparatively few teachers cover sets, relations and functions. In Ontario, British Columbia and Sweden there are core groups of classes reporting a full coverage of functions and highly variable coverage outside this core. The topic that receives the greatest coverage is algebra. For two topics there was a particularly large discrepancy between intended coverage (Figure 2-3) and implemented coverage (Figure 2-5). All of the SIMS items were intended for finite mathematics and probability and statistics, but teachers reported covering only 30 per cent.

Figure 2-5: Percentage of SIMS Population B items covered in each country, by topic, for each country.

Percent of Items Covered	Sets, Rel and Func	Algebra	Geometry	Elementary Function and Calc.	Prob. and Stat.	Finite Math
100		JPN				
90	JPN	SWE BFL FIN		JPN NZE	THA	THA JPN
80	NZE FIN BFL	CBC CON HUN FIN NZE	JPN SWE BFL	CON ENW FIN NZE	SWE JPN FIN	
70	CON THA USA	ISR THA	FIN BFL	ISR	ENW	
60	CBC SWE		SWE THA ENW	THA HUN		BFL ENW
50	HUN		CON USA	USA		NZE USA ISR
40	ISR ENW		ISR CBC		BFL USA	
30				CBC	ISR CON	CON
20					HUN CBC	HUN
10						CBC
0						
N. items	7	25	28	46	7	4
Avg. Pct.	71	87	62	78	59	61

2.6. Intended and Implemented

The curriculum analysis of SIMS is useful for an analysis of the mathematics curriculum, with or without the benefit of student achievement data. Boards, or even mathematics departments, can use the information to verify that they are teaching the topics they want and giving them the emphasis they want. Introduction of the three curriculums helps get away from the emotional discussion of the curriculum usually found in basic measurement and evaluation or curriculum foundation texts. The intended curriculum is defined by the official syllabus, but it is augmented by textbooks, course descriptions, examinations, and resource documents produced by the Ministry of Education. Boards and schools translate the guideline into a course of study, and all these documents combine in mysterious ways to define the intended curriculum.

Classroom teachers then translate the intended into the implemented. In Ontario the implemented curriculum tends to reflect the intended curriculum, because Ontario educators are prolific authors of textbooks that hew to the curriculum documented in the official guideline. In the past, it was not uncommon to find members of the curriculum guideline writing team (or their close colleagues) simultaneously writing the guideline and a textbook. Whereas Ontario is large enough to have its own textbook industry, British Columbia must import texts written for different guidelines. Quebec and the French-speaking communities outside Quebec usually rely on materials from France and Belgium which are then augmented by local distributors.

During the 1960s the curriculum reform movement was especially active in the areas of physics, chemistry, biology, and mathematics with the *new* mathematics being introduced. This was the time of the first IEA mathematics study which only partially captured the innovative spirit of the 60s. The 70s saw a stabilization period in which SIMS was designed and vigorous curriculum reforms undertaken in many educational systems. From the 1960s to the 70s Ontario changed its approach from writing a detailed syllabus to writing curriculum guidelines from which each of the more than one hundred school boards was required to detail a mathematics program. Local autonomy was granted in the specific treatment of instruction within the classrooms. As well, provincial examinations were eliminated as inconsistent with this diverse approach to instruction.

Thus, more so than in the past, when each school board in Ontario was expected to have the same implemented curriculum, it is advantageous to have the SIMS curriculum grid to compare the intended curriculum of the province with the curriculum implemented by local school boards. The diversity meant that the detailed analysis of the various curriculums and the settings were even more important. In Ontario, 50 schools were selected in which both students and teachers were asked whether the mathematics necessary to answer the achievement items had been taught (see Section 2.5), and the resulting OTL indices have been compared with the indices of intended curriculum derived from appropriateness ratings.

The difference between indices of intended and implemented content coverage are presented in Table 2-3 for Grade 8. For Ontario the weighted total is near zero, that is, the Population A items not covered are balanced by topics not in the pool. Geometry and statistics are apparently implemented less than policy intends. A typical explanation is that those preparing curriculum guidelines are unduly optimistic about what teachers are able to cover during an academic year. In Grade 8, geometry is frequently left until the end of the year, when instructional time grows short. It is common knowledge that geometry is regularly neglected by implicit design of the instructional schedule.

Although achievement is discussed more thoroughly in another section, we note in Table 2-4 that Ontario teachers are consistently overoptimistic about the performance of their students. This may stem from the same optimism that results in greater intended than implemented curriculum. The opposite is true in Japan, where teachers *under*estimated student performance, especially for geometry and statistics.

Table 2-3: Index of the Amount by which the Intended Curriculum Is Greater than the Implemented Curriculum for Population A Topics

Country	Arithmetic	Algebra	Geometry	Statistics	Measurement	Weighted Total
Belgium(Flemish)	9	22	-6	-27	-16	0
Canada/BC	9	2	-7	42	18	9
Canada/Ont.	-2	-14	27	16	-6	4
England	19	30	33	19	18	24
Finland	10	8	30	33	23	[illegible]
France	-4	11	11	-23	-30	-4
Hungary	2	-1	-1	-2	3	0
Israel	25	-3	33	-47	41	16
Japan	9	10	36	25	5	17
Luxembourg	14	21	-12	-10	-3	4
Netherlands	8	7	11	29	5	11
NewZealand	26	34	28	40	30	30
Swaziland	2	-4	-16	6	-5	-4
Sweden	20	26	13	53	28	24
Thailand	7	-10	14	21	14	8
U.S.A.	16	-6	12	28	25	14

Table 2-4: Amount by which Teachers' Predictions of Student Achievement Exceeded Actual Student Achievement.

Country	Arithmetic	Algebra	Geometry	Statistics	Measurement	Weighted Total
Belgium (Flemish)	-10*	-2	8	1	0	-1
Canada/BC	-4	4	13	7	2	2
Canada/Ont.	2	14	14	0	3	7
England	-2	-3	-10	-16	1	-5
Finland	0	0	-15	-14	-2	-6
France	-5	1	14	2	-1	2
Hungary	3	4	1	-3	8	3
Israel	3	-2	-16	-25	-3	-7
Japan	-2	-1	-13	-11	4	-5
Luxembourg	4	2	-5	-6	8	1
Netherlands	-2	1	-3	-18	2	-3
NewZealand	-6	7	-2	-15	1	-2
Nigeria	9	14	13	4	15	11
Swaziland	11	14	8	6	12	10
Sweden	-6	-3	-9	-15	-11	-8
Thailand	6	10	7	2	-2	5
U.S.A.	-4	5	11	-11	6	2

*Negative entries appear where student achievement (percent) was higher than predicted by the teachers.

Chapter 3
Attitudes and Achievement in Ontario

In this chapter we present a selection of student attitude and achievement results from the comprehensive report submitted to the Ministry of Education on the study in Ontario (see Appendix B). This will give readers the flavour of the detailed information provided by the SIMS instruments. The attitude questionnaires covered the nature of mathematics, the place of mathematics in society, students' personal feelings toward the subject and their views about calculators and computers.

These are results from students, but since, in the case of the achievement items, each student answered only those in two of the booklets, all results are averages over classes or schools. Where directly relevant, teacher reports are compared to the student responses. All the attitude and achievement results in this chapter are calculated from averages of responses to individual items over all the students in a class or a school. Class or school means are then averaged to arrive at a *score* for a topic--arithmetic, for example, or geometry.

3.1. A Close Look at Responses from Ontario's Population A Students

We will first look at achievement, focusing on fractions and on algebra. These topics were chosen because there are international results that suggest further consideration of the Ontario guideline in these areas. The international results are discussed in Chapter 4. Student responses to the attitude questionnaires will follow the achievement section.

3.1.1. Teaching and Learning About Fractions and Algebra

Nearly all teachers spend time in Grade 8 on fractions, common fractions such as 2/5ths and 3/8ths and decimal fractions such as 0.40 times 6.38. They also teach the equivalence of common and decimal fractions, such as "7 3/20 is equal to ?". Table 3-1 gives the detailed results for the 12 items making up the SIMS subset on common fractions. At the bottom of the table, the summary shows that the overall average number correct (Right) was 51 percent on the Pretest and 57 percent on the Posttest. Very few students failed to answer (Omit), so the modest gain is due to more students giving the correct answer. Looking under the column in the middle of the table marked *Change*, we see that the greatest improvement (15 percent) came on the item asking for division of fractions, a topic taught by 91.5 percent of the teachers. The first few words of the stem of the item (sometimes all of the stem) are given at the bottom of the table. One quarter of the teachers spent three class periods on this topic (not shown in this table), and only 7.4 percent of the teachers said that this had been taught prior to Grade 8!

Table 3-1: Summary of Ontario Student Responses (percent) and Teacher OTL Reports (percent) on the Twelve Items Making Up the Common Fractions Subset within the Arithmetic Topic for Population A

Item*	PRETEST Rght	PRETEST Wrng	PRETEST Omit	POSTTEST Rght	POSTTEST Wrng	POSTTEST Omit	Change	Taught Gr. 8	Taught Prior
1.	27	71	2	32	67	1	4	88.9	5.2
2.	48	46	5	58	39	3	10	79.9	11.6
3.	57	42	1	63	36	1	6	91.6	7.3
4.	27	63	10	32	62	6	5	69.6	13.6
5.	64	35	1	68	31	1	4	86.8	8.5
6.	61	38	1	65	35	1	4	92.6	5.8
7.	49	48	3	64	34	2	15	91.5	7.4
8.	51	46	3	54	44	2	4	93.0	5.3
9.	81	17	1	85	14	1	3	94.2	5.8
10.	49	48	3	56	41	2	8	92.6	7.4
11.	46	54	0	48	52	0	2	85.9	14.1
12.	55	44	1	61	39	1	5	91.9	8.1
Avg.	51	46	3	57	41	2	6		
Std. Err.	1.0	1.0	0.3	1.0	1.0	0.2	0.8		

*First Part of the Item Stem

1. Four 1-litre bowls of ice cream were set out at a party . . .
2. In the figure the little squares are all the same size and . . .
3. 2/5 + 3/8 is equal to . . .
4. Which of the points A, B, C, D, E on this number line . . .
5. Which is the closest estimate for the answer to 5-3/7 + 6-5/9? . . .
6. 3/8 - 1/5 is equal to . . .
7. (3/5) / (2/7) is equal to . . .
8. There are 35 students in a class. 1/5 of them come to school . . .
9. Which of the following is a pair of equivalent fractions? . . .
10. 1-2/5 - 1/2 is equal to . . .
11. The picture shows some black and some white marbles. Of all . . .
12. 1/2 + 1/4 is equal to . . .

Table 3-2 is exactly the same summary for the 13 items of the decimal fractions subset. Here again, almost all teachers cover the mathematics required to answer the items (with one or two exceptions) and few report that the material was covered prior to Grade 8. The very modest gains (from 47 to 52 percent overall) suggest some stubborn learning problems. The greatest gain was on the item mentioned earlier--equivalence of a common fraction to a decimal. An 11 percent gain was registered on the word problem:

> Alexandra walked from Riverview to Bridgeport, which are 3.1 km apart. During her walk she lost her watch, went back 1.7 km to find it, and then continued in the original direction until she reached Bridgeport. How many kilometres had Alexandra walked altogether when she arrived at Bridgeport?
>
> A. 1.4
>
> B. 4.8
>
> C. 6.5
>
> D. 8.2
>
> E. None of these.

Teachers strongly agreed that emphasis should be put on teaching applications of fractions. The first item on the list in Table 3-1 would appear to fall in this category. Nearly 90 percent of the teachers said they taught the mathematics, but the student success rate was near the chance level.

> Four 1 L bowls of ice cream were set out at a party. After the party, 1 bowl was empty, 2 were half full, and 1 was three quarters full. How many litres of ice cream had been EATEN?
>
> A. $3\frac{3}{4}$ L
>
> B. $2\frac{1}{4}$ L
>
> C. $2\frac{1}{2}$ L
>
> D. $1\frac{3}{4}$ L
>
> E. None of these

Students clearly knew how to detect equivalent common fractions, since over 80 percent got the item (the ninth in the list) correct on both the pretest and the posttest. Nevertheless, 95 percent of the teachers reviewed this topic, the most commonly reviewed among the common fractions topics.

Algebra topics provide a sharp contrast to fractions. Algebra is more abstract, less encountered outside of school and in most cases new to the students in Grade 8. An especially clear example is that of the *integers*, positive and negative whole numbers such as -4, 6, 0, -11 and the like. Operations with integers (addition, subtraction, multiplication and division) are taught as new content by over two-thirds of the teachers, and where not new (addition and subtraction, primarily), they are reviewed. The summary of student responses and teacher OTL reports for the five items in the integer subset is presented in Table 3-3.

Overall *Change* of 22 percent reflected the extensive teaching of this new material. The high percentage on both pretest and posttest for the last item in the list would appear to reflect successful learning of order of operations and the distributive law in prior grades, augmented by practice in multiplication. The percentage correct would almost certainly have fallen if the integers in the parenthesis had been reversed to (-4 + 6), but the gain might well have been greater.

Table 3-2: Summary of Ontario Student Responses (percent) and Teacher OTL Reports (percent) on the Thirteen Items Making Up the Decimal Fractions Subset within the Arithmetic Topic for Population A

Item*	PRETEST Rght	Wrng	Omit	POSTTEST Rght	Wrng	Omit	Change	Taught Gr. 8	Taught Prior
1.	28	67	5	37	61	2	9	85.7	5.3
2.	81	18	1	83	16	1	3	91.0	8.5
3.	60	37	3	65	34	1	5	95.3	4.2
4.	31	65	4	46	52	2	14	96.3	2.1
5.	59	40	1	65	34	1	6	63.0	13.2
6.	55	43	2	63	35	2	8	92.1	7.9
7.	39	61	0	37	63	0	2	90.5	9.0
8.	31	68	1	42	57	1	11	82.5	10.0
9.	39	59	2	41	57	1	2	66.3	[illegible]7
10.	45	49	6	51	45	4	6	72.5	6.8
11.	53	46	1	62	37	1	9	95.8	4.2
12.	19	75	6	22	74	4	4	79.0	7.0
13.	68	30	2	66	32	2	2	88.0	10.3
Avg.	47	51	3	52	46	2	6		
Std.Err.	1.0	0.9	0.2	1.1	1.0	0.2	0.7		

*First Part of the Item Stem

1. The value of 0.2131 + 0.02958 is approximately . . .
2. In a discus-throwing competition, the winning throw was 61.6 . . .
3. 0.40 + 6.38 is equal to . . .
4. 7-3/20 is equal to . . .
5. The position on the scale indicated by the arrow is . . .
6. .004)24.56
   In the division above, the correct answer is . . .
7. Which of the following is thirty-seven thousandths? . . .
8. Alexandra walked from Riverview to Bridgeport, which are 3.1 km . . .
9. The large square has area 1 square unit. The area of the . . .
10. The speed of sound is approximately 340 metres per second. . . .
11. 74.236 rounded to the nearest hundredth is . . .
12. A runner ran 3,000 metres in exactly 8 minutes. What was . . .
13. $\boxed{847.36}$ In the number in the box, the digit 6 represents . .

Table 3-3: Summary of Ontario Student Responses (percent) and Teacher OTL Reports (percent) on the Five Items Making Up the Integers Subset within the Arithmetic Topic for Population A

Item*	PRETEST Rght	Wrng	Omit	POSTTEST Rght	Wrng	Omit	Change	Taught Gr. 8	Taught Prior
1.	14	83	3	61	38	1	47	89.4	6.4
2.	40	58	2	50	49	1	10	92.6	2.1
3.	16	81	3	43	55	1	27	94.0	0.0
4.	37	55	8	57	42	2	20	89.7	3.2
5.	58	31	11	65	31	3	7	89.1	0.5
Avg.	33	61	5	55	43	2	22		
Std.Err.	1.2	1.1	0.5	1.4	1.3	0.3	1.4		

*First Part of the Item Stem

1. (-2) + (-3) is equal to . . .
2. The air temperature at the foot of a mountain is 31 degrees. . . .
3. (-6) - (-8) is equal to . . .
4. The set of integers less than 5 is represented on one of the . . .
5. -5(6 - 4) is equal to . . .

Little formal teaching of algebraic formulas is undertaken in Ontario Grade 8, in contrast to many other countries (see Section 4.1.3). Students either failed to respond (9 percent) or guessed on the following item (% choosing each alternative given in parentheses):

> Soda costs a cents for each bottle, <u>including the deposit</u>, but there is a refund of b cents on each empty bottle. How much will Henry have to pay for x bottles if he brings back y empties?
>
> A. $ax + by$ cents (12%)
>
> B. $ax - by$ cents (25%)
>
> C. $(a - b)x$ cents (14%)
>
> D. $(a + x) - (b + y)$ cents (16%)
>
> E. None of these (24%)

3.1.2. Grade 8 Students' Attitudes to Mathematics

In Section 1.1.1, teacher responses to the scale *Mathematics as a Process* were discussed (see also Table 1-1 on page 7). Students responded to the same items, agreeing with the teachers on about half of the items. In their disagreements, the students revealed a view of mathematics as slightly more static and rule-driven than the teachers. Both agreed that mathematics will change rapidly in the near future and that it is a good field for creative people, but the students were less likely to see a place for originality in solving mathematics problems. More students than teachers agreed with the statement, *Learning mathematics involves mostly memorizing*, and tended to disagree with the statement, *In mathematics, problems can be solved without using rules*. The most extreme views students had were strong agreement with *There is always a rule to follow in solving a mathematics problem*, and *Mathematics helps one to think logically*.

The scale *Mathematics and Myself* was presented to students in Grades 7 to 10 as part of the OAIP field trials in mathematics and English at the end of the 1980-81 school year and then again in 1982 in SIMS. The results were entirely consistent--students believe mathematics is important, they really want to do well and their parents want them to do well. They are uncertain about their own abilities, however, and do not express much enthusiasm for the subject (McLean, 1982b).

The eight items of the scale *Mathematics in Society* are all concerned with the uses and importance of mathematics in work and everyday life. To every item, students gave responses that restated their view of the importance and usefulness of the subject. A number of students said, however, that they could get along well without using mathematics and that it was not needed in everyday life. This is probably a genuine, if unfortunate, view of a minority of young people. For them, the mathematics studied in school appears irrelevant, and more attention to applications would not be misplaced. One hopes they stay in school and find the classes that are relevant to their needs.

Few students had access to computers in 1982, perhaps the last year that would have been true. They were generally of the opinion that "everyone" should learn something about computers, but they were undecided whether they personally wanted to learn much and many felt that they could never learn to program a computer. The survey should be repeated in a few years. Calculators were widely used, more widely than in most other countries, but students were undecided whether calculators helped them to learn. They disagreed strongly, however, with *If you use a hand calculator, you do not have to learn how to compute*.

We turn now to the responses from the mathematics specialists--from the general population at the end of elementary school to an elite sample in their last pre-university year.

3.2. Another Close Look--Population B in Ontario

As described in earlier chapters, the Ontario Population B sample was very complex. Four classes were sampled in each school, where possible, from the Grade 12 course leading to Grade 13, and one each of the three pre-university Grade 13 courses. A smaller proportion of women are usually found at this level, and this was true in Ontario--more so in algebra than in other courses. Parents' occupations revealed a disproportionate number of students from higher income groups, though as noted in Section 1.1.2, this was not as marked in Ontario as in other systems (B.C., for example). We turn first to the achievement results and then briefly to attitudes.

3.2.1. Achievement in Calculus and Algebra

As with the Population A discussion, two topic areas are presented that seem to have implications for Ontario in the light of international results. The implications will be discussed in Chapter 4, especially Section 4.2.2.

Calculus is a popular subject in Grade 13. About 62 percent of men and 44 percent of women take it. It is a traditional course, concentrating on limits of functions, differentiation, definite integrals and the calculus of functions. Analytic integration techniques (trigonometry substitution, integration by parts, . . .) are covered in about half the classes, but infinite series and partial derivatives are rarely covered. Two SIMS achievement subsets were particularly relevant to Ontario's curriculum, *differentiation* and *integration*. Summaries of student achievement will be found in Tables 3-4 and 3-5.

With one or two exceptions in each subset, teachers reported overwhelmingly that they taught the mathematics necessary to answer the questions, and achievement was related to OTL. Ontario students did reasonably, but not outstandingly, well. Only 28 percent, not many more than by chance, chose the correct answer to a standard application item--finding the maximum of a function (item 8, Table 3-4). They also had to produce the function, probably the biggest stumbling block. Evidence for an emphasis on rules for differentiation is seen in item 7, where 67 percent were able to choose the correct derivative of $exp(x^2)$.

Performance on the integration items was more variable, even discounting the three items taught by fewer than 80 percent of the teachers. Integrals of standard functions were answered correctly by nearly 80 percent of the students, and 56 percent were able to find the value of a definite integral of $x - x^{-2}$ (item 9, Table 3-5). Only 36 percent responded correctly, however, when asked to find "the area enclosed between the curve $y = x^4 - x^2$ and the x-axis" (item 4). Students were completely baffled by item 8, "Given that 3 $f'(x) = x^2 - 5$, and $f(2) = 1$, then $f(0)$ is equal to . . .", a straightforward application of the meaning of the derivative and simple integration.

Ontario students did relatively well on the algebra questions. A summary of results is given in Table 3-6. The overall success rate of 51 percent on these difficult items is good achievement. There is no information on teacher OTL because the material might be taught in any one of the three classes and students were taking various combinations of them. The average was dragged down substantially by the almost universal mistake in item 4,

> P is a polynomial in x of degree m, and Q is a polynomial in x of degree n, with $n < m$. The degree of polynomial $(P + Q)(P - Q)$ is . . .

Most students chose mn, whereas the correct answer is $2m$. Performance was good overall, however, which mathematics educators attributed to the high proportion of students who were taking the very demanding *Algebra* course in Grade 13.

3.2.2. Mathematics Specialists' Attitudes to the Subject

The attitude results can be summarized quickly. The elite sample had a view of mathematics as a dynamic, changing field, a good field for creative people. They saw it as important and useful. In contrast to the general population in Grade 8, they were more confident and felt good about their abilities. Almost none said they could get along well without using mathematics, but a few did allow that a knowledge of mathematics was not necessary in most occupations. The interesting finding was a consistent trend toward even more positive attitudes as the group became more specialized. These young adults have made their choices already, and their replies were consistent with their choices.

3.2.3. Participation by Girls

The proportion of girls in school varied somewhat from country to country. One way to compare participation by girls was therefore to calculate the *change* from Population A to Population B in the proportion of girls enrolled in mathematics classes. The general experience is that fewer girls take

Table 3-4: Summary of Ontario Student Responses (percent) and Teacher OTL Reports (percent) on the Fourteen Items Making Up the Differentiation Subset within Analysis for Population B

Item	Response Rates Right	Wrong	Omit	Teacher OTL	First Part of the Item Stem
1.	83	15	2	97	A function f is defined by $f(x) = (3x + 1)^6$. The derivative of f at x is?
2.	55	41	3	97	The graph of a function f has a point of inflection at $(a,1)$. Which of the following MUST be true?
3.	18	66	16	92	The graph of the equation $y = 3x^3 + 6x^2 + kx + 9$ is . . .
4.	66	31	3	97	The derivative with respect to x of $4/\sqrt{3x - 4}$ is . . .
5.	67	31	2	95	The velocity of a body moving in a straight line . . .
6.	47	49	4	96	At which point does the curve $y = 3x^2 - x^3$ have a local minimum?
7.	67	25	7	83	The function f is defined by $f'(x) = e^2, f(x) = ?$
8.	28	59	12	88	The intersection of a cylinder with a plane through its axis is . . .
9.	38	56	6	92	The function f defined by $f(x) = x^4 + 4x^2$ has a relative maximum of at . . .
10.	51	31	18	94	The curve defined by $y = x^3 - ax + b$ has a relative minimum point at . . .
11.	43	45	12	54	If $x - 2 \cos t$ and $y = \sin t$, find dy/dx in terms of t.
12.	38	48	14	95	Which of the following graphs has these features . . . ?
13.	17	64	20	39	f is an even function and is differentiable at 0. What condition must $f(0)$ satisfy?
14.	23	45	32	9	In the affine Euclidean plane, the coordinates of a moving m . . .
Avg.	46	43	11		
Std.Err.	1.3	1.0	1.1		

Table 3-5: Summary of Ontario Student Responses (percent) and Teacher OTL Reports (percent) on the Twelve Items Making Up the Integration Subset within Analysis for Population B

Item	Response Rates Right	Wrong	Omit	Teacher OTL	First Part of the Item Stem
1.	76	20	4	95	$\int (x-1)^2\, dx$ is equal to . . .
2.	54	36	10	3	The line l in the figure is the graph of $y = f(x)$. $\int_{-2}^{3} f(x)\, d$ equal to . . .
3.	23	70	7	82	The graph of the function f is shown above for $0 \leq x \leq 1$ $\int_0^a f(x)\, dx$ attains its greatest value . . .
4.	36	53	11	91	The area enclosed between the curve $y = x^4 - x^2$ and the x-axis is equal to . . .
5.	16	59	25	58	The value of $\int_0^1 dx(x^2 - 5x + 6)$ is . . .
6.	63	30	7	87	At which point does the curve $y = 3x^2 - x^3$ have a local minimum?
7.	83	15	2	94	$\int \sqrt{x-1}\, dx$ is equal to . . .
8.	19	69	12	89	Given that $3f'(x) = x^2 - 5$, and $f(2) = 1$, then $f(0)$ is equal to . . .
9.	56	33	11	85	$\int_1^2 (x - 1/x^2)\, dx$ is equal to . . .
10.	27	50	23	62	The function f is defined by $f(x) = \int_0^x \sqrt{1+u^2}\, du$. $f'(2)$ equal to . . .
11.	25	67	8	84	This figure shows the graph of $y = f(x)$, . . . The value of $\int_a^b f(x)\, dx$ is equal to . . .
12.	31	52	18	80	$\int_0^1 12x/(2x+1)^2\, dx$ is equal to . . .
Avg.	42	46	12		
Std.Err.	1.3	1.1	1.3		

mathematics every year, especially in the courses designed for specialists. An interesting finding was t the proportion *increased* in two of the 15 countries for which data were available. The proportion decli in all others, however, more than 50 percent in Hong Kong and Japan.

The proportion declined 22 percent in Ontario, close (as we would expect) to the 20 percent in Bri Columbia, but also close to the 19 percent decrease in the Flemish schools in Belgium. The USA was close at 16 percent. There was a greater decline of 27 percent in New Zealand and the Belgian Fre schools and still greater in England and Wales (37 percent) and Sweden (40 percent). Surprisin Scotland (at 7 percent) and Finland (at 10 percent) were quite different from their neighbors. In Israel, proportion of girls enrolled in mathematics classes declined only 13 percent from Population A Population B.

With this sort of variation, one does not expect any simple explanation, and as usual this expecta proved correct. There was a strong link between the size of the group taking advanced mathematics

Table 3-6: Summary of Ontario Student Responses (percent) on the Sixteen Items Making Up the Algebra Subset within Analysis for Population B

Item	Response Rates Right	Wrong	Omit	First Part of the Item Stem
1.	61	36	3	Which of the following points lies in the region bounded by the line $y = 1, y = x$, and $x + y = 6$?
2.	78	20	2	The curve defined by $y = 3x(x - 2)(2x + 1)$ intersects the x-axis only the points . . .
3.	31	59	10	A stationer wants to make a card 8 cm long and of such a width that when the card is cut into halves . . .
4.	12	80	8	P is a polynomial in x of degree m, and Q is a polynomial in x of deg with $n < m$. The degree of polynomial . . .
5.	72	22	2	Which of the following, $(x - 1)$, $(x - 2)$, $(x + 2)$, $(x - 4)$ are factors of $x^3 - 4x^2 - x + 4$?
6.	21	70	9	x and y are real numbers. The product of the matrices is commutat only if . . .
7.	15	73	11	According to the graph, $ax + b > cx^2$ when . . .
8.	74	22	4	What are all values of x for which the inequality $5x + 5/3 \leq - 2x - 2$ is true?
9.	78	14	8	The equation of line l is . . . and of line m is . . . . What is the solutio of the simultaneous equations . . .
10.	55	35	10	A certain number of students are to be accommodated in a hostel. If two students share each room, . . .
11.	54	36	10	Two mathematical models are proposed to predict the return y, in dollars, from the sale of x thousand units of an article . . .
12.	44	43	13	A piece of wire 52 cm long is cut into two parts and each part is ben to form a square. . . . How much longer . . .
13.	64	34	1	If x is a real number, then y, defined by $y = \sqrt{x^2 - 1}$, is a real numl for . . .
14.	46	52	1	If x and y are real numbers, for which x can you define y by $y = x/\sqrt{9 - x^2}$?
15.	45	53	2	Given that $a \geq 0, \sqrt[6]{a^5}$ is equal to . . .
16.	58	37	5	A freight train traveling at 50 km/h leaves a station 3 h before an express train. . . .
Avg.	51	43	6	
Std.Err.	0.9	0.8	0.5	

take mathematics, also encourage girls--just not quite to the same extent. Looking at all 15 count however, there was *no* link between participation by girls in mathematics and participation by studen schooling in general. In other words, keeping more students in school does not necessarily resu keeping more girls in mathematics. Hints in the international analyses suggested a combinatic influences, including the amount and kind of mathematics covered.

In Ontario, participation by girls was not the same in the three advanced courses offered in the pre-university year. The lowest participation was observed in the most demanding course, Grad algebra. This pattern was also observed in a 1983 science survey, which found higher participatio chemistry than in physics. Chemistry is a prerequisite for many of the postsecondary program health-related professions, programs more often chosen by girls. These same programs may encou applicants to take one Grade 13 mathematics course, and girls tend to choose relations and functior calculus rather than algebra. This is the pattern that leads to underrepresentation of women in sci and engineering careers and to a very small proportion of women among mathematicians.

3.2.4. Semestered vs. Year-long Classes

This leads us to an analysis of the mathematical learning experiences and accomplishments of whole group and how policies affect them. One of the many policies left to the local authority, or perhap the schools themselves, is the choice of full-year or semester organization. At the time SIMS was car out in Ontario, about a quarter of the secondary schools were organized on the semester system, wl courses begin in September and end in January, then begin again and end in June. In order to qualify *credit*, a course has to meet for at least 110 hours, so semester courses normally meet for double period half of the year. Such a system is popular with students who want to hold jobs during the school year, s timetables can be more flexible. Anecdotal reports suggest that the number of schools organizing on semester system is growing rapidly.

A substudy was therefore made of the characteristics and outcomes in semester versus year-l schools. The study has been published (Raphael, Wahlstrom, & McLean, 1986), so only a brief summ will be given here. Contrary to assertions in the literature about the benefits of semester organizat achievement was higher in the year-long classes, and this difference could not be explained by ot characteristics of the classes and schools (such as social class, rural vs. urban, . . .). No differences w found in the students' attitude to mathematics, but teachers did report covering slightly more materia the year-long courses.

Because SIMS was not designed for this comparison, the substudy has weaknesses that make results tentative. The design of the random sample of schools, for example, did not take semester year-long organization into account, so the sample might not be representative of either. More importa the achievement tests were given in May, by which time the year-long classes might have covered mor the syllabus than the half-year (semester) classes. Teachers did report, however, that they had covered intended material and were already engaged in review in both types of classes by the time the SI booklets arrived. The result can give pause, therefore, to those who plan to change from year-long semester organization. There are many reasons one might do so, but better mathematics achievement attitudes are not among them.

3.3. Linking SIMS Results Directly to Teaching and Learning

The Population A and Population B samples of Ontario schools were carefully drawn to be representative of the province as a whole, using school size, region and type (separate, private, French, public). Such a process does not yield samples representative of any particular board of education. The boards, however, are the administrative units within which all educational decisions of consequence are taken, under the guidance of the Ministry of Education. However clear the results of the Ontario study are, teachers and officials in the local authority do not have the thorough sample of their classrooms that makes the results inescapably relevant to their particular board.

At the encouragement of Steering Committee Chairman, Dr. H. Howard Russell, one board in Ontario repeated some of the SIMS study within their jurisdiction, administering the achievement booklets and the teacher OTL forms in all Grade 8 and Grade 13 classes. OISE provided copies of the booklets, analyzed the results and prepared a special report, under contract to the board. The outcomes were presented as school and classroom means, by achievement subset, coded so that local officials could identify schools and classes but others could not. The results were sufficiently useful that seven more boards have since done similar studies, not always in both populations. These local replications of SIMS are having profound effects throughout the mathematics programs of the boards conducting them. One board has now done the study in two school years.

The pattern seems to be this. The results go to the Mathematics Coordinator and the Research Director (if any), who discuss them with senior officials and Trustees (those elected to govern the authority). The Mathematics Coordinator then goes to every school, showing them where they stood relative to other schools in the board and to other countries. Officials have reported that they revised their teacher professional development plans to give attention to problem areas identified in the study and made changes to their courses of study (the detailed version of Ministry guidelines at the school level).

As the designers of SIMS expected, the description teachers and officials get of what they *actually* teach is itself of great interest, because they have never had such a systematic monitoring of their implemented curriculum. The officials have no trouble comparing this with their intended curriculum, which they know inside out. The achievement results usually confirm what teachers and officials expected, but the occasional surprise has again made the exercise worthwhile. A secondary school that had always been a leader with an excellent reputation failed to excel as they thought they should, bringing on a reappraisal of their program. All this has been possible, incidentally, without any publicity, giving the schools time to consider and act on the voluminous information without having to react to the oversimplified generalizations the media needs for its coverage.

Chapter 4
Bringing Mathematics Teaching and Learning Together

In previous chapters the study has been described--participants, curriculum analysis and attitudes. Results in Ontario were presented in some detail. In this chapter some results of particular interest are presented and their implications discussed in an international context.

4.1. Ontario's Mathematics Program at Grade 8

Recall that the definition of Population A in SIMS was "the grade in which the modal age is 13". There are two important research decisions lurking here. First, we have been concerned with the students in a particular grade of school--that is, with students in a particular set of classrooms, studying a particular program of mathematics. We are interested in how mathematics is taught and how it is learned. An alternative would be to study the achievements of the population of 13-year-olds, but that would mean, in most countries, sampling across perhaps three or four different grades, and the linkage to classroom instruction and curriculum would be lost. Second, the grade in which 13-year-olds are found is a point in the educational system of most countries when virtually all the age cohort is still in school, still taking mathematics and usually being given a common mathematics curriculum. Subsequently, students begin to stop taking mathematics, take different kinds of mathematics courses--or they stop going to school.

The definition of Population A worked out well in Ontario, because Grade 8 is a point in the system when virtually all the age group attends school and takes the same mathematics course. In the next grade in Ontario there is a major shift in mathematics education because most students transfer into secondary schools and begin to receive differentiated instruction in mathematics. After the next year the attrition from mathematics, and from school, begins.

The international perspective of SIMS shows us how Ontario's treatment of this critical juncture in mathematics education compares with what is done in other educational systems. It forces attention on some implicit and explicit curriculum choices and on the consequences of those choices. We will first focus on Ontario and the seven other countries that participated in the full SIMS Population A study, with beginning-of-year and end-of-year testing.

4.1.1. Tracking and Streaming

An important focus in the analysis of the international Population A data has been on the radically different approaches that countries make to the grouping of students for mathematics instruction. Some countries group, or stream, students by ability and others (Ontario, for example) have a policy against streaming. For the eight countries who tested at the beginning and end of the year it is possible to analyze the grouping practices empirically. The first technique to be used will be an examination of the variance in mathematics test scores (the spread or dispersion of the test scores) at the beginning of the school year, breaking that variance down into:

1. **variance between schools**--because students beginning Grade 8 in some schools score higher on the average than students in other schools. This between-school variance might reflect different neighborhood mixtures of students' socio-economic backgrounds, or it might correspond to quality differences in earlier mathematics programs (Grade 7 and before).

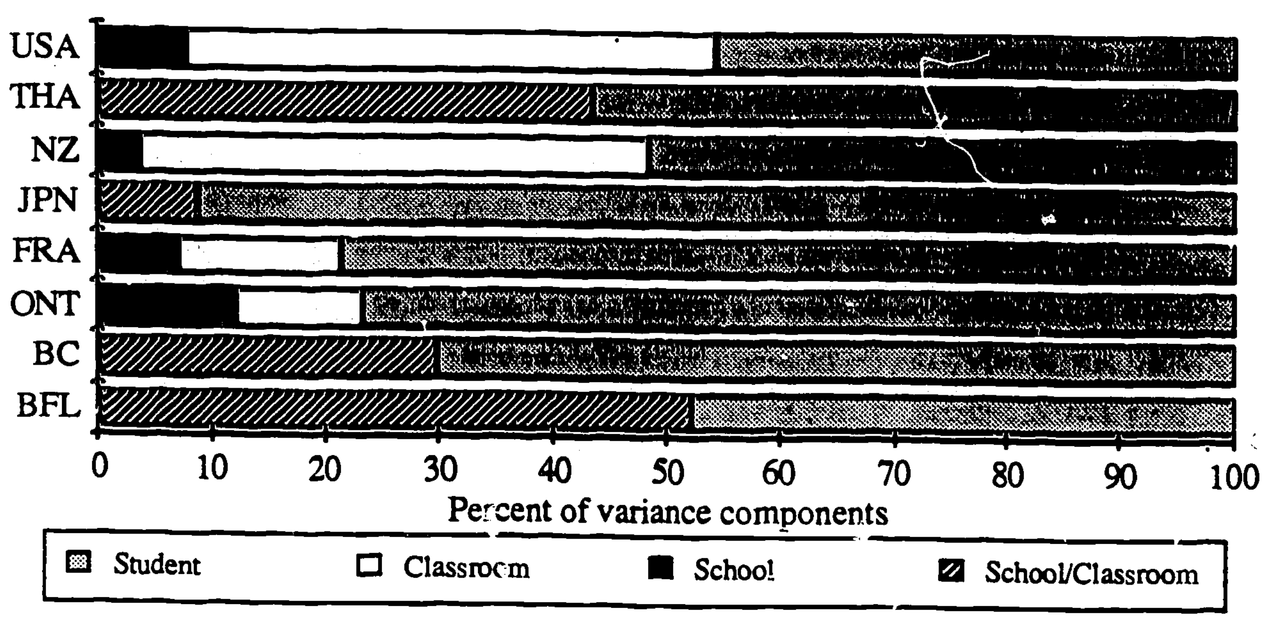
2. **variance between classrooms, within schools**--because right at the beginning of the school year, the mean mathematics achievement of classrooms normally range from high to low. In countries where streaming is practiced at the Grade 8 level, this variance should be large, and it is (in England, France and Hong Kong, for example). This variance should be low in Ontario, and it is, but not so low as in Japan.

3. **variance between students, within classrooms**--because students within a classroom will inevitably show a great variation in their initial mathematics ability and accomplishment. This variance is high everywhere, even where streaming is practiced.

Of particular interest is the proportion of variance that is found at each of the three levels--schools, classes and students. The three proportions of variance are called *components*. An analysis of the three components of variance was made on the one measure available from individual students--beginning-of-year scores on the core test. Recall that the core was an omnibus collection of different mathematics topics and would not be useful in an analysis of classroom achievement. Some items were included because they were in the first study. It can serve to describe general mathematics background at the beginning of the year.

The results are depicted in Figure 4-1, for which two technical points must be made. First, in four of the countries, only one classroom was sampled per school, so the estimates of school and classroom variance cannot be separated. Second, the variances have been estimated for *true scores*--scores for which the errors of measurement due to specific mathematics test item interactions have been discounted.

Figure 4-1: Proportion of variance in pretest scores that can be attributed to schools, classrooms and students



The large white space in the middle of the bar shows that two of the countries, New Zealand and the USA, have made a clear educational choice in favor of differentiating students at this grade level. In the case of the USA, E. Kifer has studied how tracking determines and limits content exposure in the USA.

formal curriculum differentiation (remedial, regular, enriched, and algebra classrooms may occur in the same school) largely determines content exposure and continuing mathematics opportunities. In New Zealand, the streaming is apparently more general and based on ability, but there is still a common curriculum. Japanese schools are extraordinary. Because of the sampling plan in Japan, school variance cannot be separated from classroom variance, but it is evident that both must be tiny. The schools and classrooms at this grade level in Japan are homogeneous in the extreme. The Belgium Flemish case is unusual in that they already have differentiated school systems at this grade level--academic, vocational and the like. This results in large variation between schools.

Ontario occupies a middle ground so far as tracking and streaming goes. There is about 10 percent of the variance between schools, and another 10 percent between classrooms within schools. It is education policy in Ontario that there should be no streaming or tracking in Grade 8, so the relatively small, yet statistically significant, amount of classroom variance indicates some departures from policy. One consequence of the relative *lack* of tracking may be that there is less opportunity for specialized instruction at either end of the ability continuum. Some argue that in a homogeneous system it is difficult to provide special help and remediation for the poor mathematics students and the best mathematics students may be held back. When streaming begins in Grade 9, the difference is huge. A provincial survey (McLean, 1982a) found that the difference in achievement between General and Advanced streams *within* Grades 9 and 10 was greater than the gains in average achievement from Grade 7 to Grade 10.

In spite of their successes, the Japanese are concerned about their schools and have launched a major reform effort. Bypassing the Ministry of Education, Prime Minister Nakasone named an *Ad Hoc Commission on Educational Reform* in 1984. The chairman is the former President of Kyoto University, and the membership is a cross section of Japanese society. In their second report, they found "rigidity, uniformity, and closedness . . . a tendency to impose excessive controls on students". The system has "made wastelands of children's minds". Students are not taught to think independently; they are not allowed to develop "distinctive personalities or the ability to govern themselves".[11] The Commission suggested very early that the qualities required of Japanese in the coming century should be broadmindedness, creativity, independent mind, and self-consciousness as a person in the international community.[12] It would appear that Japanese schools might not be so homogeneous in the next decade.

The Ontario decision to have homogeneous mathematics instruction at Grade 8 was likely motivated more by social concerns than pedagogical ones. Does the apparent slippage into differentiation of classrooms within schools imply a perceived or practical difficulty of dealing with the full range of adolescent students in homogeneous classrooms?

4.1.2. Content Differentiation

The variance analysis in Figure 4-1 is based on *pretest* scores, that is, scores collected at the beginning of the school year and hence not influenced by instruction carried out during Grade 8. It therefore shows just the grouping practices. Now we will look at another way in which an educational system varies the mathematics learning opportunities of students--differentiation of the mathematics *content* presented during the school year.

Information about content of instruction was collected from the teachers of the same classrooms for which the students were sampled. In Ontario this meant 197 teachers in 130 schools. As described in Section 2.5, each teacher was given, near the end of the school year, a complete set of achievement items (180) and asked to:

1. estimate what percentage of the students would likely be able to answer the item correctly,

2. indicate whether the "mathematical content necessary to answer the question" had been taught or reviewed during the year, and

3. indicate, in the case the content had *not* been taught or reviewed, whether it was taught in a previous year, a subsequent year, or not in the curriculum at all.

These responses were used to calculate an opportunity to learn index (OTL), which in its simplest form indicates whether the teacher reports that an item was taught or reviewed or, if not, whether it would have been covered in an earlier year. A positive report means that the student had an opportunity to learn the mathematical content and might be expected to get the right answer. The OTL variable can be averaged over all the items in a particular content area, such as arithmetic or algebra, and can be regarded as a characteristic of a classroom (how much OTL the students had) or even a province (how much algebra is taught in Ontario).

In Figure 4-2 the distribution over classrooms of the OTL information of the classroom teachers is displayed, and this is differentiated both by the eight countries and by five principal content areas. Each distribution is shown as a *boxplot*: the box shows where the middle 50 percent of the classroom means are, with a bar at the median level; the lines go up and down to what we might regard as the "normal" extent of the distribution beyond the central part, and then the x's and o's mark extreme values.

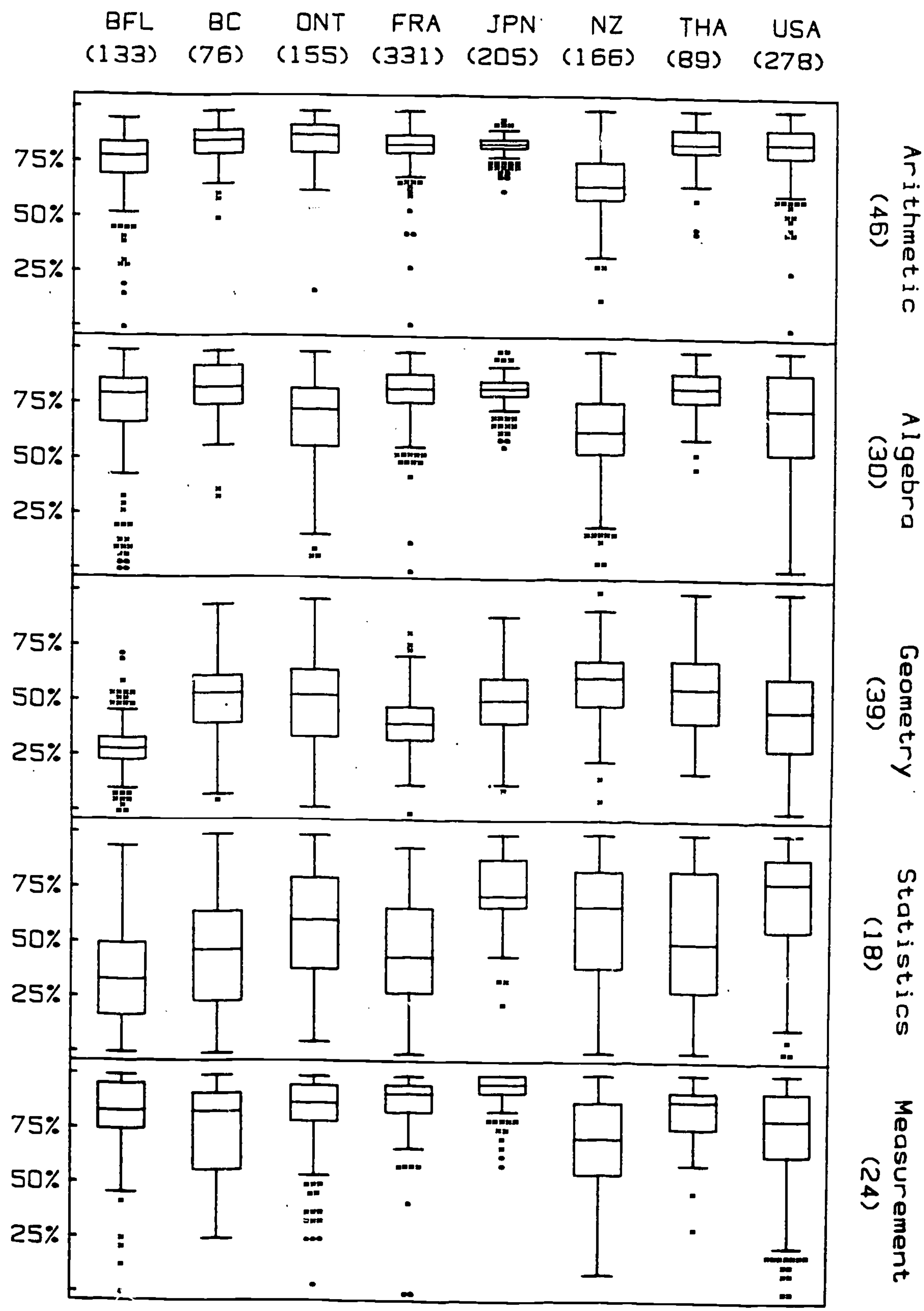
One finding evident in Figure 4-2 is that countries differ in the profiles of the mathematics content that students have (on the average) an opportunity to learn. Ontario, together with New Zealand and the USA, shows a lower opportunity to learn algebra, not a surprise since many algebra topics are optional at Grade 8. There is wide variation everywhere in the amount of geometry that is taught, and we know that this is complicated by the different kinds of géometry being taught, especially in Belgium and France, where many of the international test items were considered inappropriate. There is even more variation in the amount of teaching of statistics. For both geometry and statistics, Ontario is among the countries that teach rather more. In fact, we can see that with the exception of algebra, Ontario has one of the more extensive mathematics programs.

The boxplots also show the variation in OTL across classrooms. Except for arithmetic, the variance in OTL is generally high. For example, we can see that in Ontario, the geometry OTL is below about 30 percent for a quarter of the classrooms and above about 60 percent for another quarter (these figures are marked by the top and bottom of the box, the 75th and 25th percent points, respectively). As noted in Chapter 2, this much variation is possible even when classrooms are all using the same mathematics curriculum because of optional topics and latitude for teachers to make choices. The question then arises, if we have a lot of classrooms with low coverage of geometry and algebra, what are they studying? Are they mostly reviewing arithmetic content from previous years? The SIMS design did not produce an answer to this question, since the focus was on the SIMS items rather than on the teachers' curriculum as a whole.

4.1.3. Knowledge and Learning

Analysis and interpretation of achievement findings has to be quite cautious in view of the international differences already evident in the composition of the mathematics classrooms and the curriculums as indicated by opportunity to learn. Technical difficulties also arise when we consider achievement results, because they are based ultimately on the test-taking abilities and practices of the students, and these are not even approximately constant across the eight countries. For example, the

Figure 4-2: Boxplots of Distribution over Population A Classrooms of OTL by Content Area and Country



percentage of *omitted* responses varies from 1 to 5 percent in Thailand to 5 to 50 percent in France. Furthermore, we want to differentiate mathematical *knowledge*, which represents the accumulation of eight years of schooling, experience outside of school, common sense, and the like, from mathematical *learning* or *growth* which we can see by comparing achievement at the beginning and the end of the school year.

One way to study growth is to look at (a) *learners*, students who did not know the answer to a question at the beginning of the school year but who did give the correct answer at the end of the year, and (b) *forgetters*, students who answered correctly at the beginning but then answered incorrectly at the end of the year. In Figure 4-3, the learner and forgetter percentages, averaged over students and items, are presented for each country and for three major content areas--arithmetic, geometry, and algebra. The percentage of learners shows us in what countries there is evidence of growth, while the percentage of forgetters warns us of the possible effects of students getting the right answer by guessing. Forgetting might also be decline in achievement in older content, of course, or it might be mislearning.

First, we see relatively little learning in arithmetic. As seen earlier, all countries reported very high OTL in arithmetic, but a more detailed examination of the OTL responses, and other responses by teachers to detailed questions about their mathematics instruction, revealed that the teaching in arithmetic for the most part is *reviewing old content*, that is, content such as fractions that had been taught earlier.

Second, we see relatively little learning in geometry, with the notable exception of Japan. But geometry again is difficult to interpret because the OTL is relatively low and the correspondence to national curriculum content is irregular.

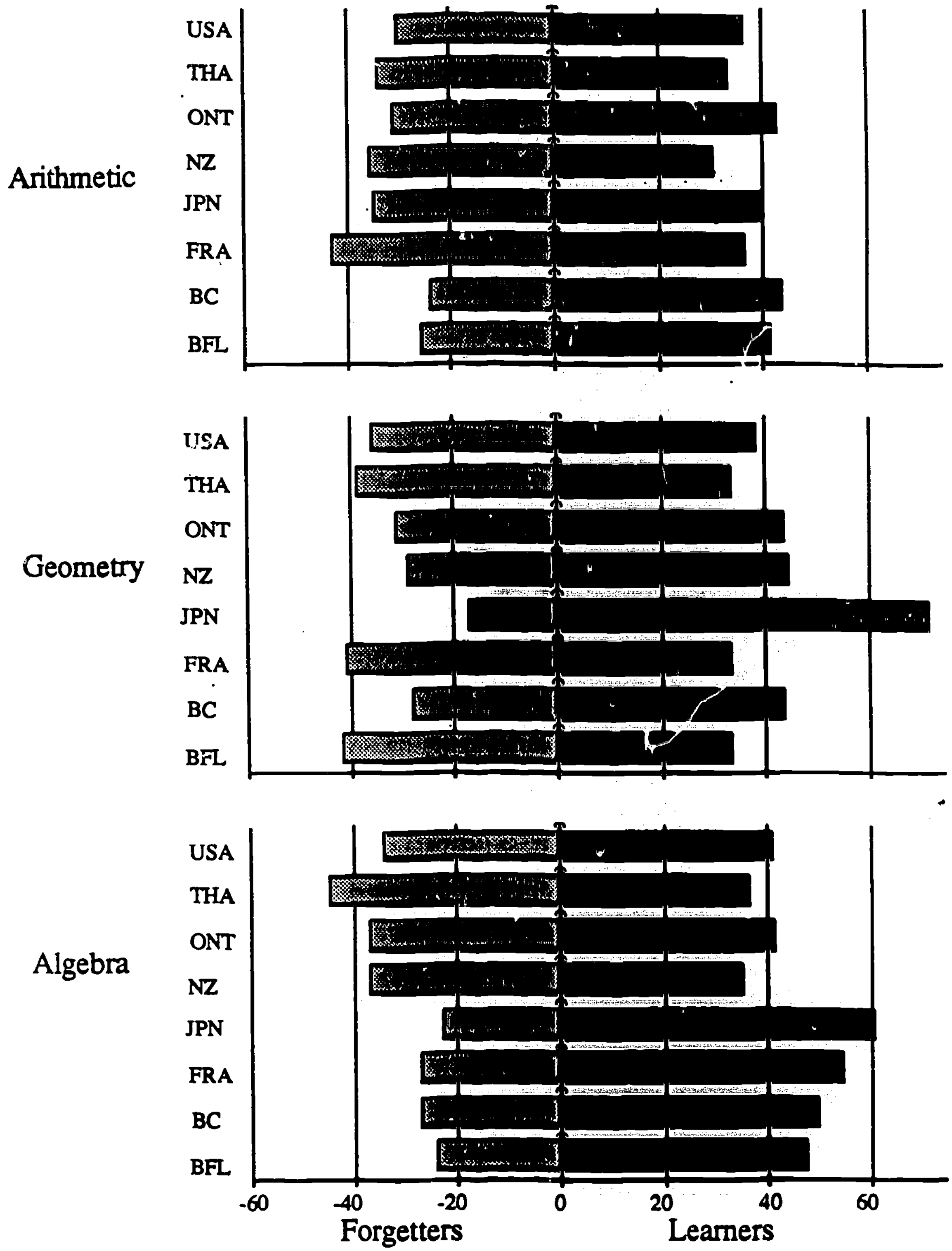
Finally, in algebra, we see a substantial amount of learning taking place in four countries, but not in Ontario. Ontario does not ask for much algebra, it does not stream students into algebra, and it does not get very much in the way of algebra achievement. In the light of the success that some other countries have in introducing algebra earlier, we should ask whether Ontario curriculum would be strengthened by enriching the algebra content in Grade 8. It is apparent that a good deal of time is spent, with little effect, reviewing arithmetic skills such as addition of fractions that were covered in the previous year or over several previous years.

4.2. Ontario's Advanced Senior Mathematics Program

The definition of Population B, "students in the last year of secondary education who are taking advanced mathematics as a substantial (five hours a week) part of a program leading to post-secondary education", means that we now shift our attention from a universal mathematics program to an educational program for the mathematics elite.

International comparisons in mathematics achievement at this level become especially difficult because there are enormous differences across the 12 countries for which data were collected in the nature of the student populations and, to some extent, in the content of the mathematics instruction. While comparing achievement is difficult, the process is still worthwhile, because it calls our attention to the crucial issue of how many students get how much mathematics.

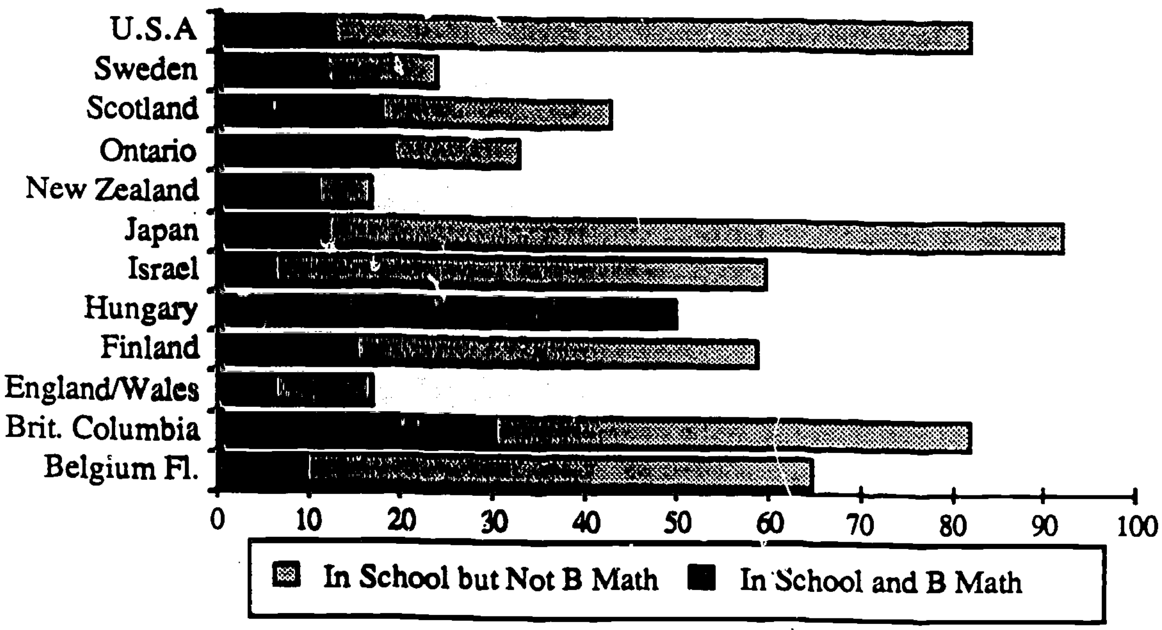
Figure 4-3: Double barcharts of Percentage of Forgetters and Learners in Three Content areas



4.2.1. Selectivity and Specialization

Since we are discussing the last year of secondary education, countries differ in the first instance in the proportion of the age cohort that survive to that level. Countries then differ again in the proportion of the surviving populations that can be considered to be mathematics specialists. This is depicted in Figure 4-4, where the total age cohort represents 100 percent, and we can see the proportion of mathematics specialists and the proportion of students in school.

Figure 4-4: Barcharts of Population B Compared to Age and Grade Cohort



The full range of participation rates is evident across the 12 countries, from the very selective systems in England and New Zealand to the unselective systems in the U.S.A, Japan, and British Columbia. The Ontario data could be misleading for someone not familiar with the system, since the decision to include students taking two or more Grade 13 mathematics courses created a special subgroup of the university-bound population. Virtually all the students are from Grade 13, while the USA and British Columbia students are from Grade 12.

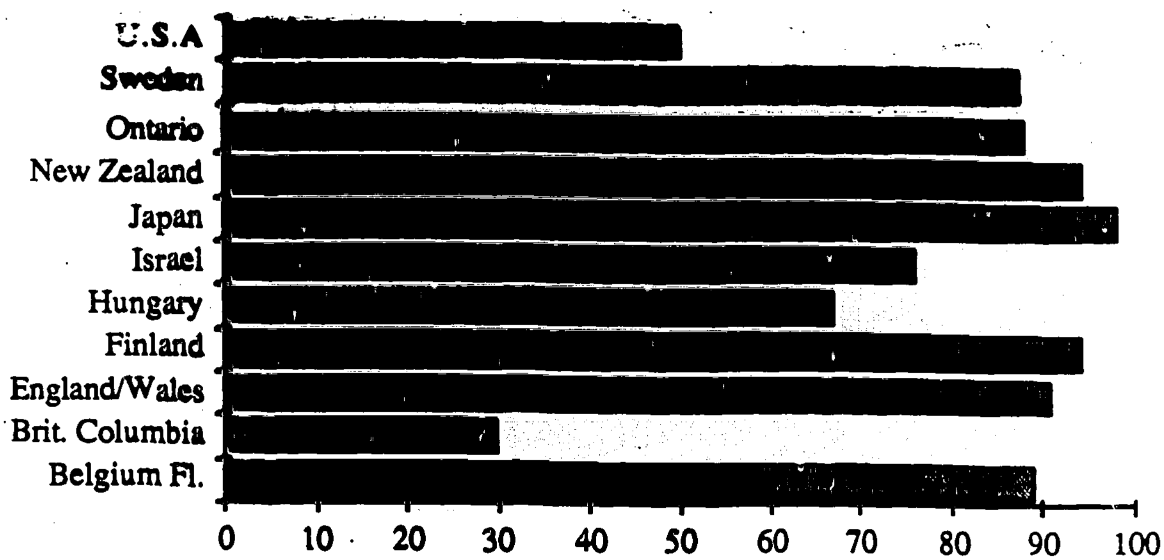
In terms of mathematics specialization, considered as a percentage of the age cohort or as a percentage in the school grade, we see tremendous variation. In Hungary, only half the age cohort survives to the final year of high school, but they all take advanced mathematics (which in Hungary means calculus). In British Columbia, a high percentage of the age cohort takes advanced mathematics, but as we shall see later, this does not include any calculus. In the other countries with high *school* participation rates, there is a compensating low *mathematics* participation rate. Ontario is again difficult to interpret and compare, but there is clearly a high rate of mathematics specialization.

These data are from 1982-83, and with the new OAC regulations now going into effect one wonders what the effect will be on Ontario's international ranking in mathematics participation.[13] It is obvious that there are radically different philosophies and policies around the world about participation in senior secondary and about mathematics specialization, and this must have an influence on the technological depth and breadth of youth in society.

4.2.2. The Content of Senior Mathematics

The mathematical content of the senior mathematics program is of interest mainly to mathematics educators, and particularly to the university educators who receive the students from the secondary schools. In Figure 4-5 we show the OTL results for *analysis*, which includes elementary functions and calculus. Note that this is based on students who are mathematics specialists, and the OTL results are not discounted for participation rates.

Figure 4-5: Barcharts of Opportunity to Learn in Population B Mathematics



One hears complaints that entering university students are not adequately prepared in mathematics, that they need to relearn what they have studied in high school. A particular issue in mathematics is the utility and efficacy of the instruction in calculus. In Ontario, virtually all mathematics specialists take calculus in high school. This is also true in most other countries, with the notable exceptions of the USA and British Columbia. It is perhaps appropriate to consider those exceptions in light of the revisions that have now been made in Ontario's mathematics program. In British Columbia, calculus is simply not available to students, and the senior mathematics program is a rather thorough course in algebra and trigonometry. The universities expect to teach calculus. In the USA, there is an unusual *national* program in advanced placement calculus; this is offered by some schools to their best students, and with a national testing program, universities grant advanced placement to successful students. Ontario students study calculus in Grade 13 and then study it again in first-year university. Their performance was poor on the SIMS trigonometry item subset.

Is it really appropriate to have so much emphasis on calculus in a system that has such a high participation rate? It has been argued that this is the reason calculus is retaught in the universities, and that time could be better spent in the high schools consolidating pre-calculus mathematics or on other topics altogether. The Project Committee drawing up the new mathematics curriculum guideline has responded to this argument and to the SIMS results by creating new courses (see Note 13), with some topics shifted and given more emphasis. Trigonometry, for example, has been moved from the relations and functions course (which will be dropped) and made a core topic in calculus. There is a new course, finite mathematics, covering matrices, combinatorics, and probability and statistics.

4.2.3. Training the Mathematics Elite

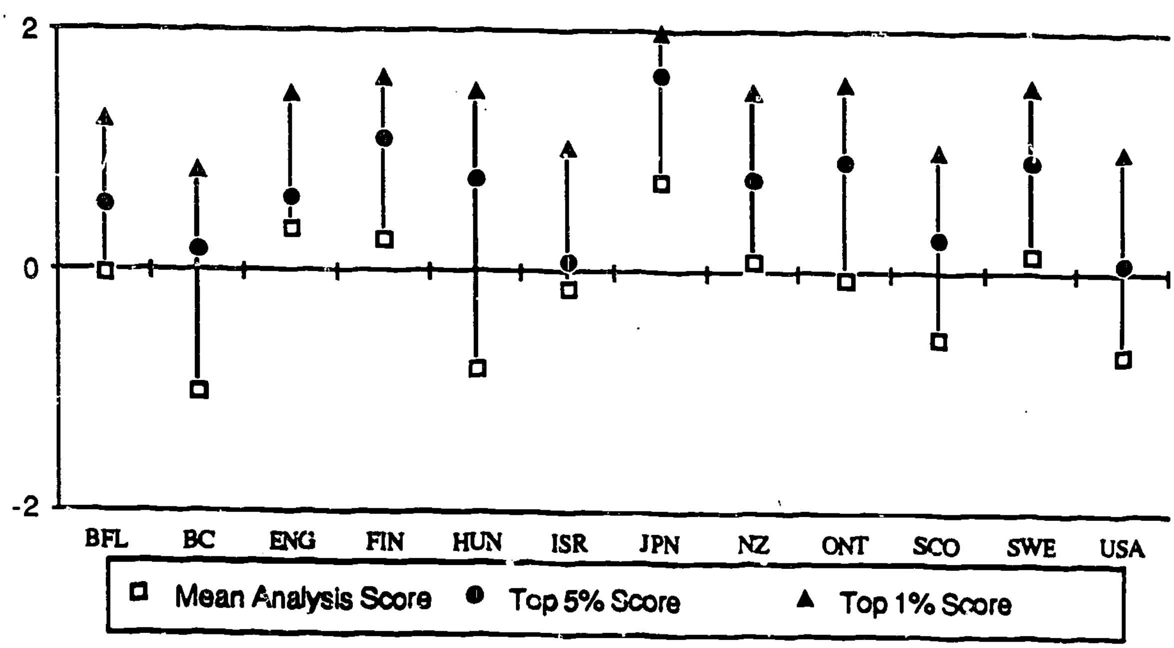
The large differences in participation rates and content of mathematics instruction make it impossible to do sensible international comparisons of achievement related to instruction. If you try to educate a large fraction of the age cohort, it seems inevitable that the average achievement will fall. We can still make comparisons, however, among the very best mathematics students. Presumably, every country strives to give adequate opportunities for these students to learn mathematics, and it is well known that high-level mathematical ability often emerges at a young age.

In a complex analysis made across the 12 countries (Miller & Linn, 1985), a scoring system was devised and then used in combination with the age cohort participation rates to estimate three scores:

- the mean score in *analysis*, based on all the students who took the test,
- the test score corresponding to the top 5 percent of the age cohort, and
- the test score corresponding to the top 1 percent of the age cohort.

Although the first score is very much a function of the participation pattern in the countries, it can be argued that the latter two scores are comparable across countries. We have to assume that the top participants score higher than any non-participants. The results of this analysis are depicted in Figure 4-6.

Figure 4-6: Line Chart of Mean Population B Achievement Levels in Analysis and Estimated Achievements of Top Percents of Age Cohort



As expected, lower mean scores were found in the countries with higher participation rates, but the country-to-country ranking for the 1st and 5th percentile scores gives quite a different story. Clearly, Japan wins the elite competition; the top 5 percent of its students do better than the top 1 percent of students anywhere else. Ontario's elite does very well, comparable to the elite in, for example, New Zealand, which is more selective, and to Hungary, which is less selective. In fact, there seems to be little connection between the selectivity of the system and the performance of the elite.

4.3. Summary and Conclusions

Ontario's participation in the Second International Mathematics Study has yielded an amazingly detailed monitoring of the Grade 8 and the Grade 12/13 mathematics program. Availability of the international results has now made it possible to produce this first public report, a very small sample of the results and implications available. Curriculum committees in the Ministry of Education have had access to the full report for some time and have made use of it in guideline revisions. One hope is that this sample will encourage more mathematics educators to dig into the full report. Better still, with OISE's help they can dig into the data to answer questions the OISE team did not address.

As noted at the beginning of this report, the evidence is strong that Ontario schools could offer more mathematics content than they do at the Grade 8 level and that a large majority of the students could learn and profit from it. There is evidence from the variation among classes that some schools may already be doing this, in opposition to the spirit, if not the letter, of provincial regulations. The very considerable amounts of time spent reviewing and reteaching some arithmetic concepts does not yield results proportional to the time spent, so some change should be made. Either better methods have to be found or else the time should be spent on other topics where more progress can be made. The following quotation from the new mathematics guideline for Grade 8 (section on *Whole Numbers and Decimals*) illustrates that this message is getting through (Ministry of Education, 1985).

> The work on place value consolidates and extends that of Grade 7. Computational skills should be reviewed and practised in the setting of applications and problem solving with an appropriate use of estimation and mental computation to anticipate the reasonableness of results (p. 34).

The general area of algebra was suggested for more attention, though this need not be abstract and formal treatment. Working through realistic problems involving money and measurement, using calculators and computers, can bring in algebra concepts and still give practice on arithmetic operations. Drill on arithmetic worksheets devoid of any real world context might well be banned from the intermediate division. Here is the introduction to the section, *Variables, Formulas, and Equations*, in the new guideline.

> Number patterns, arithmetic problems, and measurement formulas should be used to review the use of algebraic notation. The emphasis should be placed on the interpretation of algebraic expressions and equations as generalizations of arithmetical expressions and equations. Informal but systematic methods of solving equations should be consolidated (p. 36).

There was good news in the fine performance of Ontario's top students. The top 5 percent and 1 percent do well in comparison to other countries, in a system where a high proportion of the age group is in school and a high proportion of those takes mathematics. The SIMS achievement items were not such as to stretch the very best students to the fullest (in any country), but it was encouraging, nevertheless, that Ontario can provide opportunity to many without disadvantaging the top few.

That said, there are reasons to be wary of the future. Grade 13 students learned a lot of very elementary calculus that they may have to go over again in university, and they achieved only mediocre results in pre-calculus topics such as trigonometry. It remains to be seen whether a slight shift in the provincial guideline can bring about measurable improvement. Ontario students did well in algebra. What will happen with the demise of Grade 13 and the advent of the OACs? If the Calculus remains elementary and thrives and the new Algebra and Geometry withers, it would not bode well.

The OACs are said to be advanced courses, and in many respects they are. An advanced topic in the intended curriculum but absent from both the implemented and achieved was complex numbers. Every teacher said the topic was taught in someone else's class, and the students said they had never heard of complex numbers. In the new guideline, *Complex Numbers* is a core topic in Algebra and Geometry--and nowhere else. At least the intended curriculum is clear.

Finally, SIMS taught us a lot about how we should study mathematics education. Attention to the three curriculums proved to be a major strength. Never again should we be content with just the achieved curriculum. The longitudinal study in Population A (beginning- and end-of-year testing) brought home the necessity for measures of growth if we are to relate achievement to teaching. A test score at one point in time is hopelessly confounded with all prior learning, even for new topics, and such a score tells us little or nothing we can use to improve teaching. A good example was the Ontario Mathematics Achievement Test (OMAT), included as a national option because there were scores available going back to 1968. It was encouraging to learn that the average score had remained constant over 15 years (Wahlstrom, Raphael, & McLean, 1986), but beyond that it seemed to have little to tell us.

SIMS relied entirely on multichoice achievement items, at least partly because it was felt no other type could be used in a comparable way internationally. There were enough problems with the multichoice items to make one question their practical superiority but, more than that, their substantive limitations are just too great. If we are to learn whether students can solve problems, we have to give them some problems to solve and not give them a few answers from which to choose. Even a small number of constructed-response items would have added enormously to our understanding of the achieved curriculum internationally and nationally.

Ontario has already profited from its participation in SIMS--the separate reports have been discussed and the materials have been used to good effect at the local and provincial levels. Seven boards have repeated a large part of the study in their jurisdiction, with perhaps more to come. The survey of mathematics as actually taught and the achievement results closely tied to the teaching have been studied carefully and used to make changes in the official intended curriculum and locally in professional development. With the publication of this report, a larger audience can see what has been achieved. Perhaps the valuable outcomes are just beginning to appear.

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Appendix A

Participants in the Second International Mathematics Study

"Country"	Degree of Participation	Extras
Australia	Replicated First Study	
Belgium (Flemish schools)	Pop. A Longitudinal Pop. B Posttest	
Belgium (French schools)	Pop. A and B Posttest	
Canada (British Columbia)	Pop. A: core pretest + posttest Pop. B: posttest only	None
Canada (Ontario)	Pop. A, Longitudinal Pop. B, posttest only	Pop. A: Small group given posttest only as control. Pop. B: Ont. Math. Aptitude Test added.
Chile	Withdrew before testing	
Dominican Republic	Pop. A, longitudinal (started late--not in international reports)	
England and Wales	Pop. A and B Posttest	
Finland	Pop. A and B Posttest	
France	Pop. A Longitudinal	
Hong Kong	Pop. A and B Posttest	
Hungary	Pop. A and B Posttest	
Israel	Pop. A and B Posttest	
Ireland	Curriculum analysis only	
Ivory Coast	Withdrew, no data	
Japan	Pop. A Longitudinal (special pretest) Pop.B Posttest	
Luxembourg	Pop.A Posttest	
Netherlands	Pop. A Posttest	

New Zealand	Pop. A Longitudinal Pop. B Posttest	
Nigeria	Pop. A Posttest	
Scotland	Pop. A and B Posttest	
Swaziland	Pop. A Posttest	
Sweden	Pop. A and B Posttest	
Thailand	Pop. A Longitudinal Pop. B Posttest	
United States of America	Pop. A: Longitudinal Pop. B: Longitudinal	Posttest only control sample

Appendix B
Reports Published or To Be Published on the Second International Mathematics Study

International Volumes

Results will be published in three volumes by Pergamon Press in 1987.

Travers, Kenneth J. & Westbury, Ian. *Volume I: Analysis of the International Mathematics Curriculum.*

Garden, Robert A. & Robitaille, David F. *Volume II: Student Achievement in Twenty-Two Countries.*

Burstein, Leigh, Schwille, John, Cooney, Thomas J., Robin, Daniel, Robitaille, David F. & Travers, Kenneth J. *Volume III: Student Growth and Classroom Processes in Lower Secondary School.*

Selected National Reports

Crosswhite, F. Joe, Dossey, John A., Swafford, James O., McKnight, Curtis, C. & Travers, Kenneth J. (1985) *Second International Mathematics Study: Summary Report for the United States.* Champaign, Illinois: Stipes Publishing Company.

National Institute for Educational Research. National Report of Japan (in Japanese. A very short English summary is available for each volume):

- *Mathematics Achievement of Secondary School Students--Second International Mathematics Study.* Volume I, September, 1981.
- *Mathematics Achievement and Associated Factors of Secondary School Students--Second International Mathematics Study.* Volume II, March 1982.
- *Mathematics Achievement and Teaching Practice in Lower Secondary Schools (Grade 7)--Second International Mathematics Study.* Volume III, March 1983.

Robitaille, David F., O'Shea, Thomas J. & Dirks, Michael K. (1982) *The Second International Mathematics Study: The Teaching and Learning of Mathematics in British Columbia.* Victoria, B.C.: British Columbia Ministry of Education, Learning Assessment Branch.

Ontario Reports on SIMS

McLean, Leslie D., Raphael, Dennis & Wahlstrom, Merlin W. (1986) *Intentions and Attainments in the Teaching and Learning of Mathematics--Report on the Second International Mathematics Study in Ontario, Canada.* Toronto: Ontario Ministry of Education.

Raphael, D., Wahlstrom, M. W., & McLean, L. D. (in press). School structure and its relationship to student attitudes and achievement in mathematics. *International Review of Education.*

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Raphael, D., Wahlstrom, M. W., & McLean, L. D. (1984). Intentions and attainments in the teaching and learning of Ontario grade 8 mathematics: results from the second international study of mathematics. *Ontario Mathematics Gazette, 22* (2).

Raphael, D., Wahlstrom, M. W., & McLean, L. D. (1984). Intentions and attainments in the teaching and learning of Ontario grade 12/13 mathematics: results from the second international study of mathematics. *Ontario Mathematics Gazette, 22* (3).

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Notes

[1]A list of Ontario and International reports is presented in Appendix B. Readers wanting a full account of the study will want to consult the international volumes or the national reports. Appendix A contains a complete list of participants and the extent of their participation. Only a brief summary of findings is presented here, the primary purpose being to put the Ontario results in international perspective.

[2]Canada chose not to participate in the planning and implementation of the first study. The reasons appeared to be more social and political than any judgement of the quality of the plan.

[3]Swaziland's average age was 15.7 and Nigeria's 16.7, owing to special circumstances that result in many older students attending elementary school.

[4]Occupations were coded using the Pineo-Porter scale (Pineo, Porter, & McRoberts, 1977). For details, see the Ontario national report (McLean, Raphael, & Wahlstrom, 1986, Appendix VI). For the international analyses, adjustments were made to the scales in an attempt to make them more comparable. These *adjustments* resulted in some differences between the international and the national percentages. In Ontario Table 2-1, for example, the percentage of fathers in unskilled and semi-skilled jobs was given as 30 percent, whereas the international report sets the Figure at 36.6 percent. The 30 percent would still be high.

[5]*Eligible* students are those meeting the definition of Pop. B. In every country but one (Hungary), the retention rate was lower than the percentage of the *age group* still in school. The lower figure was used in all calculations involving retention rate in the SIMS study.

[6]*Mathematics Retention Rate* is Population B as a percent of the total age cohort. *Mathematics Participation Rate* is Population B as a percent of the grade cohort. *General Retention Rate* is the grade cohort as a percent of the age cohort.

[7]The synthesis was done by a Harvard Researcher, Helen Featherstone, and reported in the *Times (London) Educational Supplement.*

[8]This is something of a tautology, since the *mathematics* under consideration might well be called *school mathematics*--taught and used in school but rarely used anywhere else by most people. This is in contrast to *folk mathematics*, techniques for solving numerical and measurement problems by means not taught in school but learned and used in practice.

[9]One of 14 pages in the *Algebra* classroom processes questionnaire. Teachers were asked on page one whether the topic was (a) Taught as new content. (b) Reviewed and then extended. (c) Reviewed only. (d) Assumed as prerequisite knowledge and neither taught nor reviewed. or (e) Not taught and not assumed as prerequisite knowledge.

[10]Items had a mean difficulty across countries in the interval 0.4 to 0.9, a range of difficulties across systems and a mean discrimination of at least 0.3. Items of particular interest in this study were those judged likely to show growth over the year and those testing computational skills, estimation and approximation, proportional reasoning, items for which a calculator would be useful and "new math" items from the first study. A few items were included, chiefly linear algebra and geometry, because the Belgian national committees said these areas were important to their curriculums and were not adequately tested.

[12] *Newsletter* of the National Institute for Educational Research, Vol. 18, No. 1 (March 1986), page 4.

[13] For readers outside Ontario, OAC stands for Ontario Academic Course, courses which are part of a plan introduced in 1985 that will enable some students to complete secondary school and qualify for university in four years rather than five. Initially, most Grade 13 courses will be given as OACs, including the three mathematics courses--Algebra, Calculus and Relations and Functions. New courses are being developed, and by 1989 the three mathematics courses will be Algebra and Geometry, Calculus, and Finite Mathematics.